University of Glasgow

Adam Smith Business School

WORKING PAPER SERIES

Prudential Fiscal Stimulus

Alfred Duncan and Charles Nolan

Paper No. 2024-03
February 2024

Prudential fiscal stimulus*

Alfred Duncan[†] Charles Nolan[‡]
University of Kent University of Glasgow

This version: February 2024

Anticipated stimulus policies enacted perhaps in response to a crisis can motivate precautionary behaviour during the preceding expansion. Ex post stimulus can be ex ante prudential. Prudential fiscal stimulus both speeds up economic recoveries, and prevents crises from occurring in the first place. Prudential fiscal stimulus policies can be simple: a wage subsidy simple rule conditioned on real output can generate sizeable stimulus in downturns while improving precautionary incentives in good times. Prudential fiscal stimulus improves welfare, even in the absence of traditional aggregate demand externalities. Such policies should be implemented rapidly following a shock, and withdrawn more quickly than its dissipation.

JEL Classification: E32, D52.
Keywords: Macroeconomics, Fiscal Stimulus, Incomplete Markets.

*All errors are our own. This work was supported by UKRI Grant Ref: ES/V015559/1. We are very grateful for helpful feedback from and discussions with seminar and conference attendees at the University of Auckland, the University of Kent, the University of Wellington, the Bank of Lithuania, the Reserve Bank of New Zealand, the 2022 Money Macro Finance Society Annual Conference, the 2022 Scottish Economic Society Annual Conference and especially Campbell Leith.

[†]Corresponding author. Email: a.j.m.duncan@kent.ac.uk
[‡]Email: charles.nolan@glasgow.ac.uk

Introduction

In crises policymakers may be compelled to intervene in exceptional ways, supporting households and workers, firms, industries and overall economic activity (King, 2016, p.96). That happened in the 2008/9 financial crisis, more strikingly still during the Covid pandemic and happened once again as energy prices soared following the Russian invasion of Ukraine. However, policymakers also try to stimulate economic and financial activity in less extreme situations too; demand management policies, labour insurance and work-sharing subsidies, and the so-called ‘Greenspan put’ are obvious examples. Whilst views will differ on the net benefit of specific policy interventions, there is agreement that they distort incentives. In particular, elevated moral hazard seems to be the inevitable cost of both systematic and exceptional interventions. The purpose of this paper is to demonstrate that there are stimulus policies that encourage precaution in good times. We refer to such policies as *prudential fiscal stimulus*. The example we give is of a countercyclical wage subsidy. Even though the underlying principle of prudential stimulus is more general than our specific example, as we discuss below that does not mean all interventions are prudential.

On reflection, that such policies might exist in principle should perhaps not be too surprising. Greenwald and Stiglitz (1986) established that in economies with incomplete markets and imperfect information competitive equilibria are typically constrained inefficient. Moreover, Arnott and Stiglitz (1986) showed theoretically how one might design Pareto-improving tax policies in such economies: government should tax commodities exacerbating moral hazard and subsidize those that alleviate it. However, these authors focus on allocative efficiency and do not investigate the potential for such insights to inform the design of aggregative fiscal or financial regulatory policies as we do here. Again, hence our nomenclature: *prudential fiscal stimulus*.

Prudential fiscal stimulus policies increase welfare even in the absence of aggregate demand externalities. In our benchmark model, the *only* channel through which stimulus can increase welfare is through improved incentives for precautionary behaviour in good times.[1] Thus, initially we set aside nominal rigidities as a source

[1] We develop a New Keynesian version of our model later in the paper.

of aggregate demand externalities and focus on the efficiency of stimulus policies in the presence of a contracting friction between firms and investors. In an extension we also show that the same insights go through if instead the model is recast as one where the financial friction lies between households and banks.

Stimulus policies are prudential when they complement precautionary behaviour by firms (including banks) in good times. Our primary example is a simple rule for the design of a wage subsidy. That rule conditions the wage subsidy on real GDP; when GDP is low, the wage subsidy is high, and vice versa. Labour is a complement to firms' inside wealth, and the countercyclical wage subsidy increases the marginal value of firms' inside wealth in downturns. That encourages firms to take precautionary actions in good times, ensuring that they can take advantage of the wage subsidy in downturns. In the banking extension, the wage subsidy similarly increases the marginal value of bank equity in downturns. In short, the prudential stimulus nullifies the usual concerns with government intervention; instead of blunting beneficial precaution, the wage subsidy boosts productive capacity in downturns by encouraging firms (banks) in good times to accumulate more inside wealth (net worth) than they otherwise would.

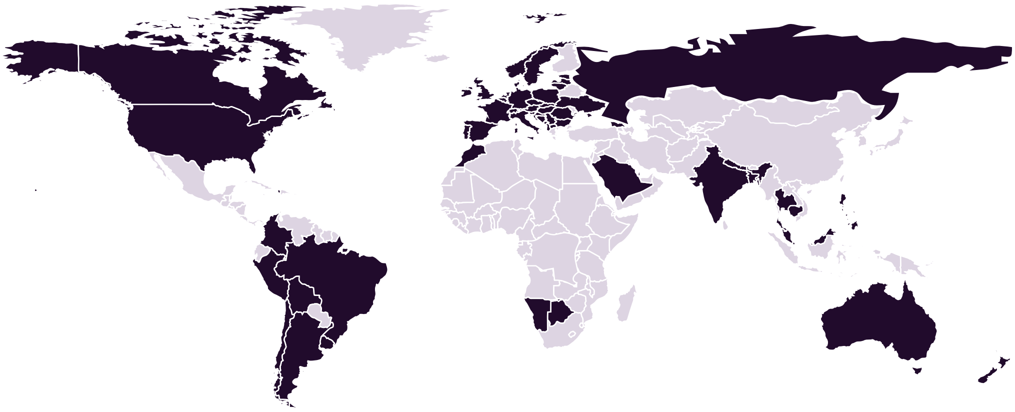
Wage subsidies are not the only prudential fiscal stimulus policy in our model but they are of much current interest given the widespread use of related schemes during the Covid-19 pandemic. Based on ILO (2020) and IMF (2021) reports, we calculate that 72 countries (the regions shaded in Figure 1) have used either existing or new wage subsidy and job retention subsidy schemes as part of their economic response to the Covid-19 pandemic.[2]

Of course, promotion of wage-subsidy schemes is not new. Phelps (1997) has argued for subsidizing low-wage workers' employment via a subsidy to firms for every

[2]By May 2020, wage subsidy and job retention schemes supported about 50 million jobs across the OECD, about ten times as many as during the global financial crisis. As time went on more workers were incorporated into schemes. For example, in the UK, France and Italy at the peak of the schemes a third or more of their workers were included. That said, we do not mean to imply that all countries adopted similar schemes. Schemes varied substantially across countries in terms of duration, generosity and design. The wage subsidy we model is inevitably a somewhat stylized version of those actually implemented. See https://www.oecd.org/coronavirus/policy-responses/ supporting-jobs-and-companies-a-bridge-to-the-recovery-phase-08962553/ for some details on design. Linden et al. (2021) look at the design and impact of different wage subsidy schemes on various metrics: The Structure and Incentives of a COVID related Emergency Wage Subsidy Jules Linden, Cathal O'Donoghue, Denisa M. Sologon. See also our discussion later.

Figure 1: Countries with new or existing wage subsidy schemes

during the Covid-19 pandemic



Sources: The International Labour Organization (2020), the International Monetary Fund (2021), authors' calculations. Wage subsidy programmes are defined as any programme that provides wage support to employers to retain workers in employment.

low-wage employee on the payroll. He argues that this kind of scheme would have far-reaching social and economic benefits. And interestingly from the point of view of the present paper, part of that benefit would be a likely boost to the equilibrium level of employment (e.g., as quit rates diminish).[3]

However, concerns around the impact these schemes might have on the incentives for firms to adjust to the shock of the pandemic have never been far from policymakers' and economists' thoughts. And so the policy challenge in withdrawing

[3]See the wide-ranging analysis in Phelps (1997). The promotion of these schemes goes back still further. They were also proposed during the Great Depression in the US and the UK, by highly respected economists. For the US, Galbraith (1975) notes that in 1933 Irving Fisher proposed (unsuccessfully) to President Roosevelt government deficit financing of a scheme that would pay directly to private employers an interest free loan rising in the number of workers added to the payroll. The wider context of his discussion is that of a perceived failure of purely monetary open market operations to reverse sharp contractions in activity. This subsidy scheme, as with others in the US to which Galbraith therein refers, were time-limited as a way to encourage take-up and boost private spending. In the UK, probably the most high-profile proponent for the potential of wage subsidies was Arthur Pigou, see Pigou (1933). The book is somewhat technical for its time and some historians of thought suggest contemporary economists may not have given the book as much attention as they might have otherwise. That said, numerous substantive reviews, by the likes of Seymour Harris, Roy Harrod and Paul Sweezy, were published. In any case, he essentially appears to endorse targeted wage subsidies as a means to boost the economy. See also the likely more influential paper by Kaldor (1936). He was even more enthusiastic than Pigou as to the potential for wage subsidies to cure what he called 'general unemployment.'

this support is often presented in terms of balancing those disincentive effects with the benefits of employment and social protection. We do not deny that such a tension exists in practice. It is also worth noting that some of the challenge in coping with a pandemic, say, resides in the fact that some sectors of the economy have to be shut-down in whole or in part; for a time at least subsidising those sectors' wages may be less effective in boosting aggregate output. However, in helping to stimulate other sectors of the economy, regardless of a pandemic or similar sector-specific shock, wage subsidies may be effective as stimulus.[4] Our argument then is that the design of any support package is crucial to the nature and extent of disincentive effects and that 'well-designed' support need not be as damaging as typically thought; indeed, in principle it can be welfare improving, as we demonstrate.

Model overview and intuition

The core model consists of a population of risk averse entrepreneurs, and a population of risk averse worker-rentier households. Entrepreneurs combine their net wealth with borrowed wealth and wage labour to produce consumption goods. However, entrepreneurs' ability to defray idiosyncratic production risk is limited because of the information asymmetries that disrupt the contracting process. In other words it is costly for any lender to observe the entrepreneur's idiosyncratic productivity and that costly state verification limits risk sharing. The result is that informational asymmetries limit the pledgeability of income. [5]

In downturns inside wealth is scarce and earns high expected returns. But moral hazard distorts this price signal; when inside wealth is scarce, the shadow costs of moral hazard are high, and optimal external finance contracts concentrate production risk with entrepreneurs. Entrepreneurs, thus exposed to increased production risk, discount the high expected returns.

The upshot is that entrepreneurs absorb more aggregate risk, through their choices in production, labour and capital markets, than under the socially optimal allocation

[4] See discussions in Woodford (2022) and Guerrieri et al. (2022) pertaining to stimulus versus insurance policies, specifically in the context of Covid.

[5] In the banking version of the model in Appendix K, banks combine deposits from households with banks' equity making state contingent loans to firms, who have idiosyncratic production technologies. Just as informational asymmetries limit the pledgeability of income in the core model, in the banking version banks are unable to diversify away idiosyncratic risk.

of aggregate risk. And this is so, even when aggregate risk markets are open. The competitive equilibrium is constrained inefficient and there is scope for policies to internalise the social costs of that excessive risk taking.

More broadly, there are welfare gains available from policies that complement entrepreneurial prudence in periods of high income, including stimulus policies. The main requirement for a welfare improving policy in this environment is that it should increase the marginal value of entrepreneurial wealth in downturns, thereby encouraging prudence in the good times.

The main result of the paper is Proposition 1, which derives a closed-form expression for the optimal state contingent labour subsidy (or tax) in a non-linear DSGE model supporting a range of underlying financial frictions. Optimal wage subsidies increase on impact from shocks that tighten financial frictions, but the size of the optimal wage subsidy is moderated by historical shocks: wage subsidies are most effective if introduced quickly in response to shocks.

The specific financial friction that we introduce for quantitative analysis is the imperfect state verification model based on Duncan and Nolan (2019) which, as discussed below, provides a foundation for the optimality of debt contracts.[6] We estimate the nonlinear model and use the estimated model to quantify an optimal wage subsidy simple rule responding only to output deviations from the steady state. We find that a wage subsidy simple rule with an elasticity on output of 1.8 maximises expected welfare gains, and reduces the expected welfare costs of business cycles by 20 percent relative to the laissez-faire counterfactual. By this measure, expected welfare gains increase for regions of the posterior where more of the persistence of the model is endogenously generated from our financial friction; welfare gains decrease when the persistence of model dynamics is driven more by the persistence of the exogenous efficiency shock process.

Our benchmark model only includes one friction, and our policy instrument acts on the labour market, a market that is subject to a number of distortions in practice. We undertake two extensions to verify that our results hold in the presence of further labour market distortions. First, we impose a 40 percent constant labour tax, which ultimately increases the optimal size of wage subsidies and the resulting welfare

[6]Our results extend to a broader class of contracting frictions. The banking model presented in Appendix K uses a simple private information friction without audits.

gains. Second, we introduce New Keynesian price rigidities, which generate a time varying labour market wedge of inefficiency. We also estimate the New Keynesian version of our model and solve for optimal monetary and wage subsidy policies, demonstrating that wage subsidies are part of the efficient policy response to shocks.

There is another reason why the New Keynesian extension is of interest. In our flexible price model, prudential fiscal stimulus is not time consistent—when the downturn arrives, policymakers have no incentive to introduce a distortionary stimulus policy and rational agents know that. However, when aggregate demand externalities are present, things are more complex. It is well known that acting under discretion, policymakers have an incentive to accommodate supply shocks, but that the rational expectation of this policy stimulus may worsen outcomes (Barro and Gordon, 1983; Kydland and Prescott, 1977). Our New Keynesian model presents a different set of time inconsistency tensions where things turn out better. On the one hand, under flexible prices the prudential benefits of stimulus are determined by the expectation of that stimulus in future downturns, to which a policymaker under discretion cannot commit. On the other hand, however, aggregate demand externalities mean that policymakers, acting under discretion, ex post will have a desire to accommodate inflationary shocks in downturns and that encourages stimulus. The distance between policies under commitment and discretion is less when there are aggregate demand frictions.

Related literature

Prudential fiscal stimulus policies can be interpreted as Arnott and Stiglitz (1986) interventions. These authors studied optimal tax in a multi-good economy with identical agents where asymmetric information resulted in moral hazard. They showed that, absent government intervention, the competitive equilibrium is usually constrained inefficient so that differential commodity taxation is optimal; government should tax those goods that exacerbate moral hazard and subsidize those that alleviate it. That is the underlying intuition behind our result too. A stimulus is incentive improving if it discourages complements of moral hazard. In our model, exposure to macroeconomic risk is a complement to moral hazard. Optimal policy diminishes this exposure, and in doing so boosts economic activity and generates a welfare gain.

This deterrence could be implemented through ex ante regulation, but also through ex post stimulus policies.

As in Arnott and Stiglitz (1986), we assume that individuals and firms are anonymous when they interact in different markets. That eliminates the possibility of constrained efficient decentralised across-market insurance bundles studied by Prescott and Townsend (1984) and the markets-for-markets mechanisms studied by Kilenthong and Townsend (2014).

Our quantitative analysis uses the imperfect state verification model of Duncan and Nolan (2019), which extends the model of Townsend (1979) to support standard debt contracts as optimal even when lenders can credibly commit to stochastic monitoring strategies. Thus, (debt) contracts in our model are privately optimal, given the information and technologies available to agents. The model, therefore, generates a macroprudential wedge of inefficiency that is closely related to Di Tella (2017) and Duncan and Nolan (2021). One important difference with Di Tella (2017) is that our model supports the financial amplification of technology shocks.

As a result of financial frictions, entrepreneurs bear residual project risk. Those same financial frictions do not rule out trade in aggregate risk with worker-rentier households. During periods of financial stress, high returns to inside wealth correspond to high levels of residual risk. Discounting these returns ex ante, entrepreneurs accept a larger share of aggregate risk in competitive allocations than in constrained efficient allocations. Ultimately, this motivates a role for what is, in effect, macroprudential policy so as to internalise the social costs of fluctuations in firm leverage. Alternatively, ex post interventions that increase the marginal value of entrepreneurial wealth in downturns can restore incentives for ex ante entrepreneurial prudence in economies without optimal macroprudential policies in place. That is what the wage subsidy accomplishes.

1 The model

The model consists of a population of firm owner entrepreneurs, and a population of identical worker households. Entrepreneurs have access to a special production technology that generates stochastic output from labour and a fixed capital factor, K the sum of entrepreneurs' and households wealth, q^e and q. Entrepreneurs and households both have risk averse preferences with respect to consumption—although the extent of risk aversion is allowed to differ across groups in our quantitative analysis.

Production y is subject to idiosyncratic shocks, $y = \theta z f(K, h)$ where θ is an idiosyncratic, firm-specific shock, z is a common total factor productivity shock, and f is a constant returns to scale production function. Information asymmetries are assumed to limit the extent to which entrepreneurs can defray risk to outside investors. Beyond information asymmetries, there are no other ad hoc restrictions on financial markets within periods—entrepreneurs could always raise additional funds from non-contingent loans, but leverage is limited by entrepreneurs' capacity for risk bearing in equilibrium. We first set out the entrepreneurs' and households' approach to aggregate risk sharing. Then we turn to the implications of the contract required to encourage households to invest in firms given the asymmetric information, costly and error-prone state verification.

1.1 Entrepreneurs

We decompose the entrepreneurs' problem into two parts. At the end of each period, after realising output and repaying any within period loans and labour contracts, entrepreneurs choose their consumption and trade in aggregate risk securities. At the beginning of the following period, aggregate risks are realised, entrepreneurs' aggregate risk securities are paid, and entrepreneurs hire labour and borrow to fund capital acquisition for production.

At the end of the period, an entrepreneur faces the following problem:

Programme 1 *At the end of the period, an individual entrepreneur solves the following problem:*

$$v^e(q^e) = \max_{x^e, c^e, q^{e\prime}} \mathbb{E}_{\Theta,S} \left\{ \log c^e + \beta^e v^e(q^{e\prime}) \right\}$$

subject to the resource constraint,

$$q^{e\prime} = R(\theta)q^e - c^e - \int_{s \in S} p(s)x^e(s)ds + x^e(s'),$$

where $R(\theta)$ is a mapping that solves the entrepreneur's within period problem.

The timing notation convention is that without loss of generality, q^e is measurable in the time t information set, while $q^{e\prime}$ is measurable in the period $t+1$ information set. Trade in aggregate risk markets is captured by the quantities $x^e(s)$, denoting the amount purchased of an asset with payoff 1 conditional upon the future state of the world being realised as state s. The current period price of this security is denoted $p(s)$. As indicated earlier, trade in securities indexed by the aggregate state are not hampered by any problem of asymmetric information; unlike idiosyncratic states, aggregate states are costlessly observed and verified by all agents. These markets are active.

Our ability to separate an entrepreneur's problem in this way depends on our use of information asymmetries only as the source of market incompleteness in our within period financial market. Collateral constraints, or similar, that restrict borrowing within period would also restrict agents' choices of aggregate risk securities x^e.

Programme 2 *At the beginning of the period, an individual entrepreneur solves the following problem:*

$$R(\theta) = \arg \max_{b^e, h^e, k^e} \mathbb{E}_\Theta \log R(\theta)$$

subject to the capital constraint,

$$Qk^e = q^e + b^e,$$

the truth-telling constraint,

$$R(\underline{\theta}, \overline{\vartheta})q^e + (\overline{\theta} - \underline{\theta})zf(k^e, h^e) \leq R(\overline{\theta}, \emptyset)q^e, \tag{1}$$

and the lenders' breakeven constraint,

$$\sum_{(\theta,\vartheta)} P(\theta, \vartheta)\left[\theta zf(k^e, h^e) - (R(\theta, \vartheta) - 1)q^e\right] \geq wh^e + rb^e + P(\underline{\theta})\kappa k^e. \tag{2}$$

The first constraint defines the value of capital contributing to an individual entrepreneur's project as the sum of their initial wealth and funds borrowed by the entrepreneur from households. Equation 1 is the incentive compatibility constraint. $R(\underline{\theta}, \overline{\vartheta})q^e$ denotes the gross income received by an entrepreneur whose productivity shock is wrongly revealed to be the high type. $R(\overline{\theta}, \emptyset)q^e$ denotes the gross income received by an entrepreneur who receives the high type productivity shock. Equation 2 is the households' participation constraint. Repayments to external financiers are contingent on the idiosyncratic shock θ as well as any audit signal obtained by the household (or bank), $\vartheta \in \{\underline{\vartheta}, \overline{\vartheta}\}$. Audit signals are distributed as follows: $P(\overline{\vartheta}|\overline{\theta}) = 1, P(\overline{\vartheta}|\underline{\theta}) \in (0, 1)$, so that high types are always correctly identified by the audit, but with some probability low types are incorrectly tagged as high types. Thus to access outside investment, entrepreneurs need to commit to overturned low reports paying out more than high reports. This induces truth-telling and is ensured by the incentive compatibility constraint. To ensure the investment of households, expected loan repayments must exceed the sum of the households' opportunity cost of funds, rb^e plus expected audit costs, $P(\underline{\theta})\kappa K$, where κK is the cost of each audit for some constant κ. It turns out that that when audit costs are sufficiently low, defaultable debt contracts with deterministic audit strategies are optimal (Duncan and Nolan, 2019).

In the extension of our model, focusing on banks/financial intermediaries, we use a different microfoundation for within period loans. The important features of within period loans for our analysis are that (1) idiosyncratic risk cannot be fully passed on to outside investors, and (2) borrowing decisions are interior, with entrepreneurs always able to borrow an additional dollar on a non-contingent basis. Microfoundations based on information asymmetries alone are well suited to generate these two features.

The next few sections focus on three key features of the competitive equilibrium. First, the entrepreneurs' aggregate risk portfolio choices are combined with those of households to examine economy-wide, competitive allocation of aggregate risk. Despite these markets being open it is shown that the financial accelerator is not closed down, even for TFP shocks. Second, some details are provided on the financial contract to show intuitively why, entrepreneurs having taken on excessive risk from a societal perspective, a wage subsidy can be an effective countervailing instrument. Finally, it is shown that the financial friction results in a wedge in factor prices–less labour and capital is hired than otherwise would be the case. Moreover, there is a strongly positive relationship between equilibrium leverage and the factor price wedge.

1.2 Households

The problem for the representative household is

$$v(q) = \max_{x,c,h,q'} \mathbb{E}_S \left\{ u(c,h) + \beta v(q') \right\}$$

subject to

$$q' = (1+r)q + wh - c - \int_{s\in S} p(s)x(s)ds + x(s')$$

Household utility is defined over consumption and labour, h. Without loss of generality, q_{-1}, q, q' are measurable in the period $t-1, t, t+1$ state spaces respectively. Households supply labour and use the income gained along with accumulated wealth, q, and the payoff from their Arrow-Debreu portfolio, to save, invest and consume. Households have access to a risk-free asset that pays off r per unit invested.

1.3 Financial market clearing

Aggregate risk insurance markets allocate risk between the household and entrepreneurial sectors. The sum of Arrow securities contingent on any possible future state of the world is equal to zero.

$$x^e(s) + x(s) = 0 \qquad \forall s \in S.$$

1.4 Intertemporal risk sharing

The result of the trade in aggregate risk markets is the following equilibrium risk sharing condition

$$\frac{\beta^e}{\beta} \frac{u^{e\prime}(\bar{c}^e)}{u'(c)} \frac{E_\theta R(\theta, s)}{1+r} = \frac{u^{e\prime}(\bar{c}^e_{-1})}{u'(c_{-1})}, \tag{3}$$

where $\mathbb{E}_\theta[R(\theta, s)]/(1+r)$ denotes the gross equity risk premium within the period, and $\lambda' := u^{e\prime}(\bar{c}^e)/u_c(c, h)$, where $\bar{c}^e$ is the aggregate consumption of entrepreneurs. In the absence of individual specific risk, Equation 3 would collapse to the traditional perfect risk sharing condition $\left(\beta^e u^{e\prime}(\bar{c}^e)/(\beta u'(c)) = u^{e\prime}(\bar{c}^e_{-1})/u'(c_{-1})\right)$. The equity risk premium is equal to the difference between entrepreneurs' expected consumption marginal utility and the marginal utility of expected entrepreneurial consumption. The derivation of Equation 3 is in Appendix A.

The interpretation of Equation 3 is important. It seems natural to intuit that aggregate risk markets would, in effect, recapitalize firms in downturns as Krishnamurthy (2003), Nikolov (2014) and Carlstrom et al. (2014) show in workhorse financial accelerator models of Kiyotaki and Moore (1997) and Bernanke et al. (1999). If aggregate risk markets did indeed act this way then financial amplification, and the necessity for remedial policies, essentially disappears. However, that is not the case here. When entrepreneurs are risk averse and optimal leverage is interior, the high return to inside wealth during a downturn is countered by high costs of production in the form of risk premia. That is, a low level of aggregate productivity combines with the risk of low idiosyncratic productivity leading entrepreneurs to value consumption smoothing relatively highly compared with financial stabilisation (that is, accruing inside wealth). In short, competitive markets in macroeconomic risk do not expunge credit cycles.[7] It is this behaviour that opens up scope for welfare-enhancing public policy, as the decentralized equilibrium is constrained inefficient: Society, though not individual entrepreneurs with access to aggregate risk markets, would prefer to go into recessions with a larger aggregate stock of inside wealth. And so, by subsidizing labour which is a complement of wealth, the authorities can move the economy closer to the constrained efficient outcome.

1.5 Financial contracts and factor prices

The equations in this section are mostly those delivered by the optimal external financing contract given costly state verification, audit errors and risk averse agents. The firm/entrepreneur and the household agree on a contract that is incentive com-

[7] In contrast with Di Tella (2017), productivity shocks are amplified through credit cycles in our model, as are interest rate shocks and cost-push shocks in the New Keynesian version of our model we present in Section 5.

patible and penalises the entrepreneur found to be lying, something that can be costly if a truthful report of low productivity is erroneously overturned. The contract also satisfies a participation constraint for the household. If entrepreneurs are going to attract household investment they will need to offer an expected return greater than r and expected audit costs. Because the entrepreneur is risk averse, a financial wedge emerges from the covariation of the entrepreneur's marginal utility and firm outcomes. That wedge is increasing in both firm specific uncertainty and leverage. It will also show up in the demand for labour (and for capital). It will also increase the equity risk premium. The derivations of (4,5,6) are contained in Appendix H.

Leverage is the ratio of expected entrepreneurial production over the opportunity cost of entrepreneurial wealth,[8]

$$l = \frac{y}{(1+r)q^e}. \tag{4}$$

where y denotes aggregate income, $y := \mathbb{E}[\theta]f(h)$.

The equity risk premium is defined as the ratio of the expected return to entrepreneurial equity over the risk free rate. The equity risk premium is increasing in the factor price wedge τ as well as the leverage ratio, a measure of the scarcity of entrepreneurial wealth,

$$\rho = 1 + l\tau \tag{5}$$

where $\tau = -\text{cov}(\Upsilon(\theta), \theta)$ and where $\Upsilon(\theta) = \dfrac{u^{e\prime}(c^e(\theta))}{\mathbb{E}_\Theta[u^{e\prime}(c^e(\theta))]}$.

Wages differ from the expected marginal product of labour by the factor wedge τ, which reflects the cost of the marginal increase in residual risk borne by the entrepreneur resulting from an increase in labour hired,

$$w = \mathbb{E}[\theta]f'(h)(1-\tau). \tag{6}$$

[8]Many related models in the literature measure leverage as the ratio of productive assets to net wealth. Our definition of leverage captures the effect of increased labour factor inputs on production risk. As such, our measure captures risk-adjusted leverage.

Factorless income can be derived from equations 4,5, and 6:

$$(\rho - 1)(1 + r)q^e = \tau y.$$

This is the income accruing to the entrepreneurs over and above the risk free rate earned on their inside wealth, and is the result of the risk premium due to the undiversifiable component of inside wealth.

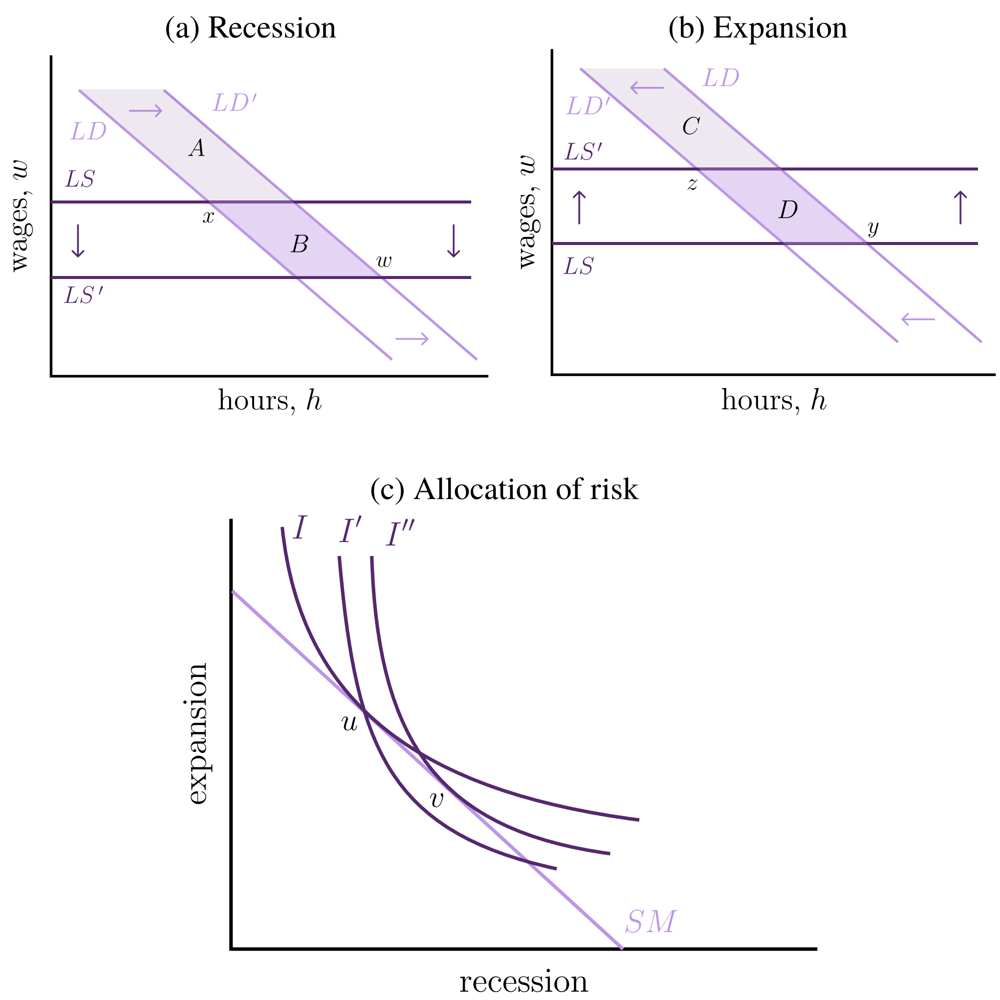
The interaction between factorless income and aggregate risk sharing determines the dynamics of leverage and financial amplification in our model. When entrepreneurial net wealth is scarce, the equity risk premium and factorless income increase, helping to restore entrepreneurial net wealth. At the same time, aggregate risk sharing (3) amplifies the procyclicality of entrepreneurial net wealth.

1.6 The effect of wage subsidies on entrepreneurial risk taking

The financial contract concentrates too much risk, from a societal perspective, on entrepreneurs. To see intuitively how wage subsidies might help, consider Figure 2 which presents a partial equilibrium analysis of the effect of countercyclical wage subsidies on an individual firm's risk taking. The firm starts at x, y, u in panels (a,b,c) respectively, taking wages as given. Consider a firm with access to aggregate risk insurance markets; the firm can insure against a recession or expansion in the next period. The labour supply schedule is initially given by LS. A firm may decrease its risk by buying insurance against a recession and selling it against an expansion. So, a small decrease in the firm's risk taking before the realisation of expansion or recession would shift the firm's labour demand from LD to LD', an increase in labour demand if the economy enters recession and a decrease in labour demand if the economy enters expansion. The firm's allocation of risk is optimal before the realisation of the aggregate economic state if they value areas $C + D = A$ after adjusting for their marginal rate of substitution across expansions and recessions.

Now, let the policymaker introduce a wage subsidy (tax) upon realisation of a recession (expansion). The labour supply schedules faced by the firm now shift from LS to LS'. A small decrease in the firm's risk taking now increases surplus by $A+B$ in the recession state which is strictly greater than C, the decrease in surplus in the

Figure 2: Countercyclical wage subsidies and firm risk taking.



Notes: The diagrams represent the partial equilibrium effects on the labour market of the introduction of a countercyclical wage subsidy (LS to LS') coupled with a decrease in firm risk taking (LD to LD').

expansion state. The value of wealth carried forward into the recession state has increased relative to the value of wealth carried forward into the expansion state, and the firm's indifference curves in the risk allocation market rotate. The firm chooses their risk allocation from their budget set, where the slope of the SM schedule reflects the relative market prices of Arrow securities contingent on the realisation of expansion and recession states reflectively. Ultimately, the firm reduces their risk taking, moving from u to v.

With the intuition of the model set out, we now set out the essential features of the financial friction and then turn to characterise optimal wage subsidy policies.

2 A quantitative model with imperfect state verification

Our choice of specific financial friction determines the relationship between the residual risk borne by entrepreneurs, their leverage, and idiosyncratic production risk. For our quantitative analysis, we assume that financial contracts are subject to a version of the imperfect state verification model, although as we explain the framework here is easily adapted to alternative financial frictions.

Optimal external finance contracts are subject to a costly state verification with imperfect, costly audits following Duncan and Nolan (2019). Importantly, this environment generates standard debt contracts as privately optimal, even when lenders can commit ex ante to ex post monitoring strategies, and even when the lender has access to lotteries. If audits were perfect, and there was no aggregate risk, the environment would be that of Townsend (1979). He showed that debt contracts would be the optimal response in such an environment so long as agents were constrained to deterministic auditing regimes. He noted, however, that a better contract might employ a stochastic auditing schedule and that conjecture was confirmed by Border and Sobel (1987) and Mookherjee and Png (1989). These authors then showed that optimal contracts, rather than being debt, resemble equity finance—repayments are contingent on marginal fluctuations in income. However, with risk-averse entrepreneurs and audits that are imperfect and may wrongly indicate productivity draws, Duncan and Nolan (2019) show that once more debt contracts can be optimal. Another way to see this is to note that equity-like contracts provide more insurance across states for the contracting parties, but given imperfect audits may make a bad situation worse for a borrower with a genuinely low productivity draw that is nevertheless tagged as misreporting. [9]

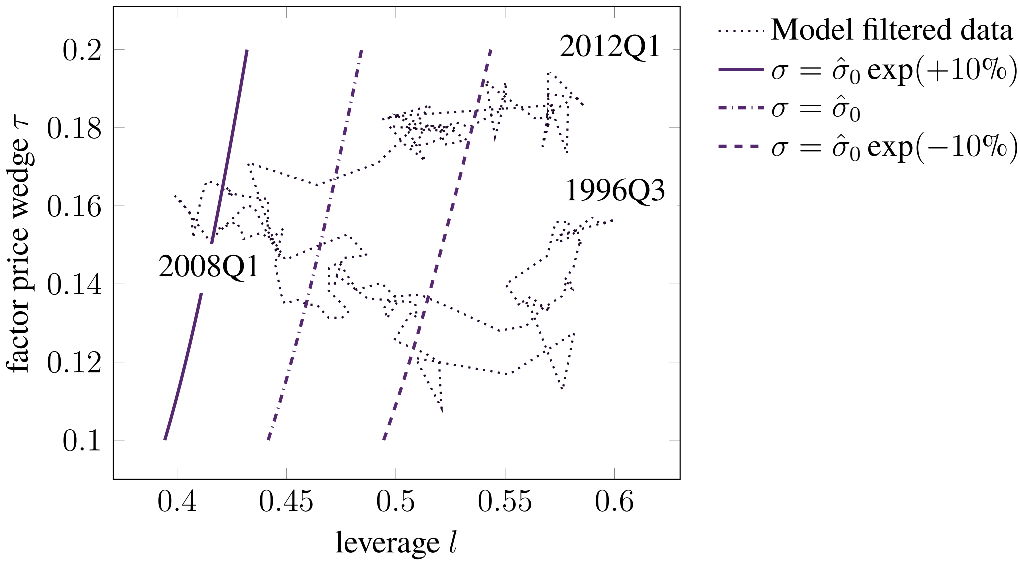
The upshot is that the contracts we study are privately optimal given the information and record keeping constraints that we impose. We do not require any

[9] Duncan and Nolan (2019) discuss in detail the literature and other requirements for the optimality of debt contracts in their imperfect audits csv model. The main additional requirement is that audit costs are not too high. That paper also fits the model to the US data.

reduced-form pledgeability constraints, which are unlikely to be robust to the policy interventions that we consider. Specifically, our model predicts that policy interventions affect the implied pledgeable share of wealth. Clearly, these effects would be missing in models where the pledgeable share of wealth is exogenous.

This model provides a tractable convex mapping from leverage l and idiosyncratic risk σ to the factor price wedge τ. Figure 3 presents this relationship in the neighbourhood of the calibrated deterministic steady state, with model filtered values derived from US data. This function, derived in the appendix, is described by Equation 7. In the neighbourhood of the deterministic steady state, the factor price wedge is sharply increasing in leverage and idiosyncratic risk.

Figure 3: Leverage and the factor price wedge



Notes: The dotted series presents the values for leverage and the factor price wedge that consistent with observed US data, filtered through equations (4) and (6). The solid, dashed, and dash-dotted schedules represent the relationship between leverage and the factor price wedge for different levels of idiosyncratic risk, given by Equation 7. All values are computed using a calibrated parameter vector. The parameter $\hat{\sigma}_0$ represents the steady state level of idiosyncratic risk.

$$\tau = \frac{\left(\sqrt{\frac{\bar{\pi}}{\underline{\pi}}} - \sqrt{\frac{\underline{\pi}}{\bar{\pi}}}\eta\right) l\sigma - (\bar{\pi} + \underline{\pi}\eta)}{2l} + \frac{\sqrt{\left[\left(\sqrt{\frac{\bar{\pi}}{\underline{\pi}}} - \sqrt{\frac{\underline{\pi}}{\bar{\pi}}}\eta\right) l\sigma - (\bar{\pi} + \underline{\pi}\eta)\right]^2 + 4\eta l^2\sigma^2}}{2l} \tag{7}$$

Here σ is the standard deviation of idiosyncratic risk, and as before, $\underline{\pi}$ is the probability of a low draw of that risk, and $\bar{\pi}$ is the probability of a high draw. Finally, η is the probability of a Type-1 audit error; that is failing correctly to identify an entrepreneur who is truly low productivity. It is worth emphasizing that Equation 7 is the only equilibrium condition in the model that depends on the assumption of imperfect state verification. Adapting the model for other agency problems (for example, hidden actions or costly state falsification) would mean replacing Equation 7, and the consumption growth equations (F1,F2) in the appendix required for welfare calculations. No further changes to model equations are necessary. Appendix K presents the alternative model where the private information financial friction applies to financial intermediary liabilities rather than productive firm liabilities, and shows that our main finding carries through to that environment.

3 Optimal wage subsidy policy under log household utility

The policymaker weights the welfare of the representative household by μ, and the value of the representative entrepreneur by μ^e. The representative entrepreneur begins at time zero with wealth equal to mean entrepreneurial wealth. Social welfare v^s is defined as follows:[10]

$$v^s := \mu v + \mu^e v^e \tag{8}$$

We apply the Pareto weights that imply that the competitive equilibrium distribution of wealth is socially optimal at the economy's steady state. These weights can be

[10] We assume that the policymaker is interested in the welfare of both groups in the economy. An alternative formulation would be to assume that the policymaker is only directly interested in the welfare of the working households, and to include a participation choice for entrepreneurs: the policymakers' weight on entrepreneurial welfare μ^e would then correspond to the Lagrange multiplier on the entrepreneurs' participation constraint.

interpreted as Negishi (1960) weights. Full details of the welfare calculations in our quantitative analysis can be found in Appendix F.

The representative household's consumption leisure trade-off becomes

$$\frac{-u_h}{u_c} = w(1+\varsigma) \tag{9}$$

The wage subsidy ς is funded by lump sum taxes, paid by the household sector. The household sector's budget constraint becomes

$$q' = (1+r)q + wh(1+\varsigma) - c - \int_{s\in S} p(s)x(s)ds + x(s') - T.$$

The policymaker faces the balanced budget constraint

$$T = wh\varsigma.$$

Proposition 1 *Assume the following utility functions:*

$$u(c,h) = \log c - \frac{h^{1+\psi}}{1+\psi}, \qquad u^e(c^e) = \log c^e.$$

a. The socially optimal wage subsidy is described by Equation 10:

$$\varsigma^* = \frac{\tau}{1-\tau}\left(1 - \left(\frac{l_0\rho - l\rho_0}{\rho - 1}\right)\frac{(1-\beta^e)}{l_0 - (1-\beta^e)\rho_0}\right). \tag{10}$$

b. Output is completely stabilised in response to uncertainty shocks, and is proportional to total factor productivity.

A proof of Proposition 1 can be found in Appendix C.

The implications of the optimal wage subsidy on equilibrium output are as follows. Under the competitive equilibrium in the absence of wage subsidy policy, real output follows

$$y = z\left(\frac{\alpha l(1-\tau)}{l - (1-\beta^e)\rho}\right)^{\frac{\alpha}{1+\psi}}. \tag{11}$$

Under the optimal wage subsidy, real output follows

$$y = z \left(\frac{\alpha l_0}{l_0 - (1 - \beta^e)\rho_0} \right)^{\frac{\alpha}{1+\psi}} \tag{12}$$

Comparing Equations 11 and 12, real output is proportional to total factor productivity in both regimes, but only responds to fluctuations in leverage and risk under the competitive equilibrium. Under the optimal wage subsidy regime, output does not respond to uncertainty shocks.

The wage subsidy also reduces consumption risk over the cycle for households. As a consequence, households' demand for aggregate risk insurance falls, lowering the market price of insurance and further encouraging firms to purchase more aggregate risk insurance. This wealth effect of the wage subsidy for households further supports the prudential channel of wage subsidy policies.

Equivalently, the wage subsidy induces higher labour supply from households. It also induces higher inside wealth accumulation on the part of entrepreneurs (because they get the benefit of subsidised labour, a complement to their wealth stock). And so, entrepreneurs buy more aggregate (consumption) risk insurance for recession times than they otherwise would because the labour subsidy makes inside wealth more valuable during such times.

3.1 Time inconsistency of optimal subsidies

It can be readily established that optimal wage subsidy policies are time inconsistent. Optimal wage subsidy policies distort economic allocations within period, with high wage subsidies in downturns lifting hours worked above the point where the resulting consumption marginal utility exceeds the total costs including the increased costs of risk bearing. The policymaker's objective in using wage subsidy policies is to manipulate agents' risk taking allocations ex ante, and the resulting distortion of production and labour supply within the current period is the cost.

Thus, in the flexible price setting optimal policies are time inconsistent: If the policymaker could signal a wage subsidy conditional upon a future downturn, and this generated a prudential response in financial allocations today, then upon realisation of the downturn there would be no incentive to implement the signalled wage subsidy. Proposition 2 formalises this result. Under discretion, where policymakers can re-set their optimal wage subsidy upon observing the net wealth that entrepreneurs bring into the period, there is no incentive to introduce wage subsidy policies, and allocations are identical to the competitive equilibrium.

Proposition 2 *Assume the risk free real interest rate r is fixed. Under discretion, the optimal wage subsidy is zero in all periods.*[11]

As is demonstrated in Section 5, commitment to wage subsidy policies is less important when the economy suffers from aggregate demand externalities. It is shown there that a wage subsidy policy response is part of the optimal policy response to a recessionary shock under discretion. The upshot is that the discretionary policy response is somewhat similar to the optimal commitment policy in that setting; aggregate demand externalities ameliorate the time-inconsistency problem that afflicts optimal wage subsidies.

3.2 Financial amplification and risk aversion

Before turning to quantitative analysis of the benefits of wage subsidies, it is worth noting that while the assumption of log utility can be analytically convenient, it is restrictive. Specifically, when households are more risk averse than entrepreneurs—a natural assumption to make—leverage decreases in response to efficiency shocks, generating procyclical financial amplification. Equation 13 presents the derivative of current period leverage with respect to current period output. When household risk aversion is greater than under log utility ($\gamma > 1$), leverage is decreasing in current period output, generating a financial accelerator effect in response to TFP (and other) shocks.

$$\frac{dl}{dy} = -\frac{l}{y}\,\frac{(\gamma-1)(l-(1-\beta^e)\rho)}{l+(\gamma-1)(1-\beta^e)\rho}. \tag{13}$$

This financial amplification is an important feature of the model, and we allow for non-log household utility in the estimated non-linear model we use for quantitative analysis.

[11] Note that the assumption of a fixed risk free real interest rate has no effect on the optimal policy described in Proposition 1.

4 Optimal wage subsidy simple rule

Having characterised the optimal wage subsidy for the model under log utility, we now relax the assumption of log utility and turn to the analysis of a simple, optimised wage subsidy rule. This analysis is motivated by some of the same considerations that motivate the study of optimised simple rules in other areas of monetary and fiscal analysis (such as the vast literature on optimised 'Taylor Rules'). The idea is essentially that in practice policymakers often act like they condition instrument feedback control on a subset of widely observed variables. For our purposes, then, we propose the following wage subsidy simple rule

$$\varsigma = -\phi_{\varsigma}(y - y_0), \tag{14}$$

where y_0 is deterministic steady state output.

One way to think of this, is that rather than the policymaker observing and reacting to individual firm leverage or the financial wedge—firm specific or economy-wide—the policymaker is 'hoping' the rule-based subsidy conditioned merely on GDP is positively correlated with that fully optimal subsidy. This rule is also loosely motivated by attempts to support the labour market during the recent pandemic. Of course, most of the schemes introduced were not new. For instance, in New Zealand, the scheme was built on earlier schemes such as the Earthquake Job Loss Cover, a payment to labour which applied both to full time and part time workers. However, although many of the schemes were not new, the extent of their application was. And in some countries–like in the UK–the (furlough) scheme was in fact new. As we observed earlier, the schemes otherwise varied substantially across countries. The US for instance channelled more of its support for labour by extending unemployment insurance. Many other countries extended work-sharing schemes, or short-time working (Kurzarbeit) as in Germany and a number of other countries. Schemes also varied substantially across countries in terms of duration, generosity and many other aspects of design. As Giupponi et al. (2022) observe, schemes may be classified by their principal focus, either on insuring workers (as in the US) or insuring job matches as in the UK and many other OECD economies. However, as these authors also observe, in practice schemes were not so discontiguous.

For instance, in the US context, recall unemployment insurance also has job match preservation features. In any event, the wage subsidy we model is inevitably a somewhat stylized version of those schemes actually implemented, but is clearly closer to schemes that emphasised job insurance (and which were in operation in many countries even before the pandemic). [12]

Figure 4 shows the welfare gains resulting from wage subsidy simple rules, using our parameter estimates described in Appendix G. The persistence of the TFP shock affects the welfare assessment of subsidies. For our estimated parameters, a wage subsidy rule with coefficient $\phi_\varsigma = 1.8$ maximises expected welfare gains across posterior draws and simulated histories, reducing the expected welfare costs of business cycles by 20.5% relative to the counterfactual of no wage subsidy policy.

While modest wage subsidy simple rules generate sizeable welfare gains for all posterior parameter estimates, we find that the magnitude of welfare gains from wage subsidy simple rules is sensitive to the value of the persistence of TFP shocks. Figure 5 presents the persistence of TFP shocks, ρ_z, and the welfare gain from the associated optimal wage subsidy simple rule, for a sample of parameter draws from the posterior. Posterior draws with lower values for the persistence of TFP shocks correspond to larger welfare gains from the fixed wage subsidy simple rule. When TFP shocks are highly persistent, the welfare costs of recessions are due more to the long tail of low productivity than to the financial acceleration of the recession itself. As a result, there is less scope for policy interventions to reduce the welfare costs of business cycles. In other words, subsidy schemes ought to be introduced quickly following a shock, but the value of the subsidy should be reduced more quickly than the shock's effects dissipate. It also seems that shocks to the financial friction also require more robust subsidies.

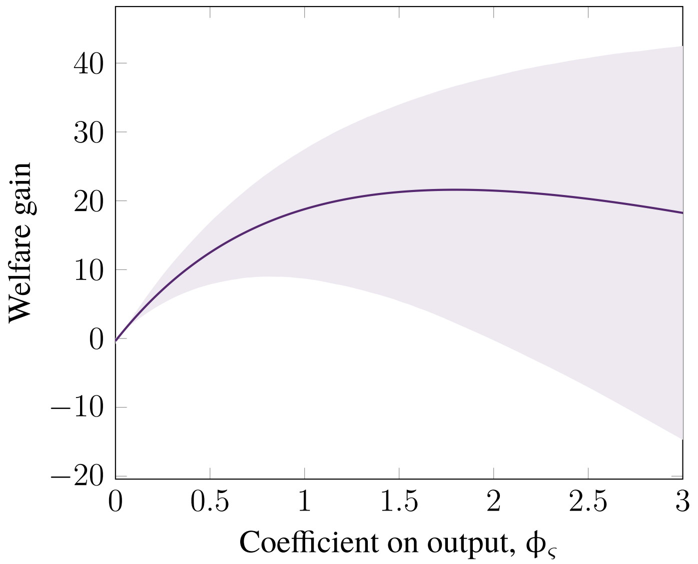
4.1 Crises and optimal simple rules

We turn now to the likely performance of the simple rule during a crisis. For this we need an example of a crisis. We draw parameters from the posterior parameter

[12] As was noted earlier, specifically in the context of a pandemic shock when certain sectors may have to shut down or scale back production, the wage subsidy has to be interpreted with more nuance. In that case one might emphasize that it can play a role as part of a wider attempt to boost aggregate demand and/or to absorb labour more quickly into sectors with scope to expand, than might otherwise be the case.

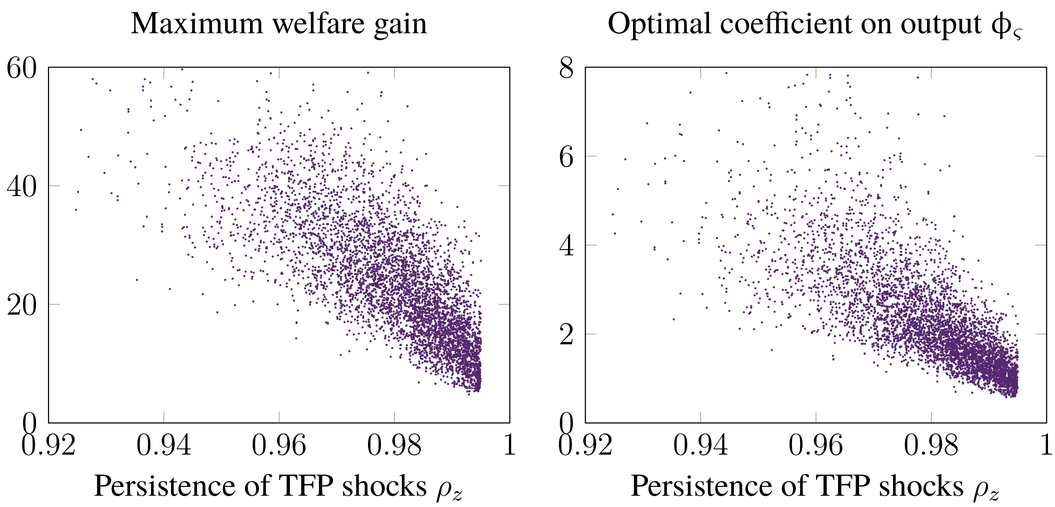
Figure 4: Estimated mean welfare gain from optimal simple rule

(Percent decrease in business cycle welfare costs)



Notes: The welfare gain is reported as the reduction in the welfare costs of business cycles as a percentage of those welfare costs in the absence of policy intervention. Shaded area indicates 90% credible interval. Parameter values are estimated under the assumption that there is no wage subsidy policy.

Figure 5: Simple rule welfare gains and the persistence of TFP



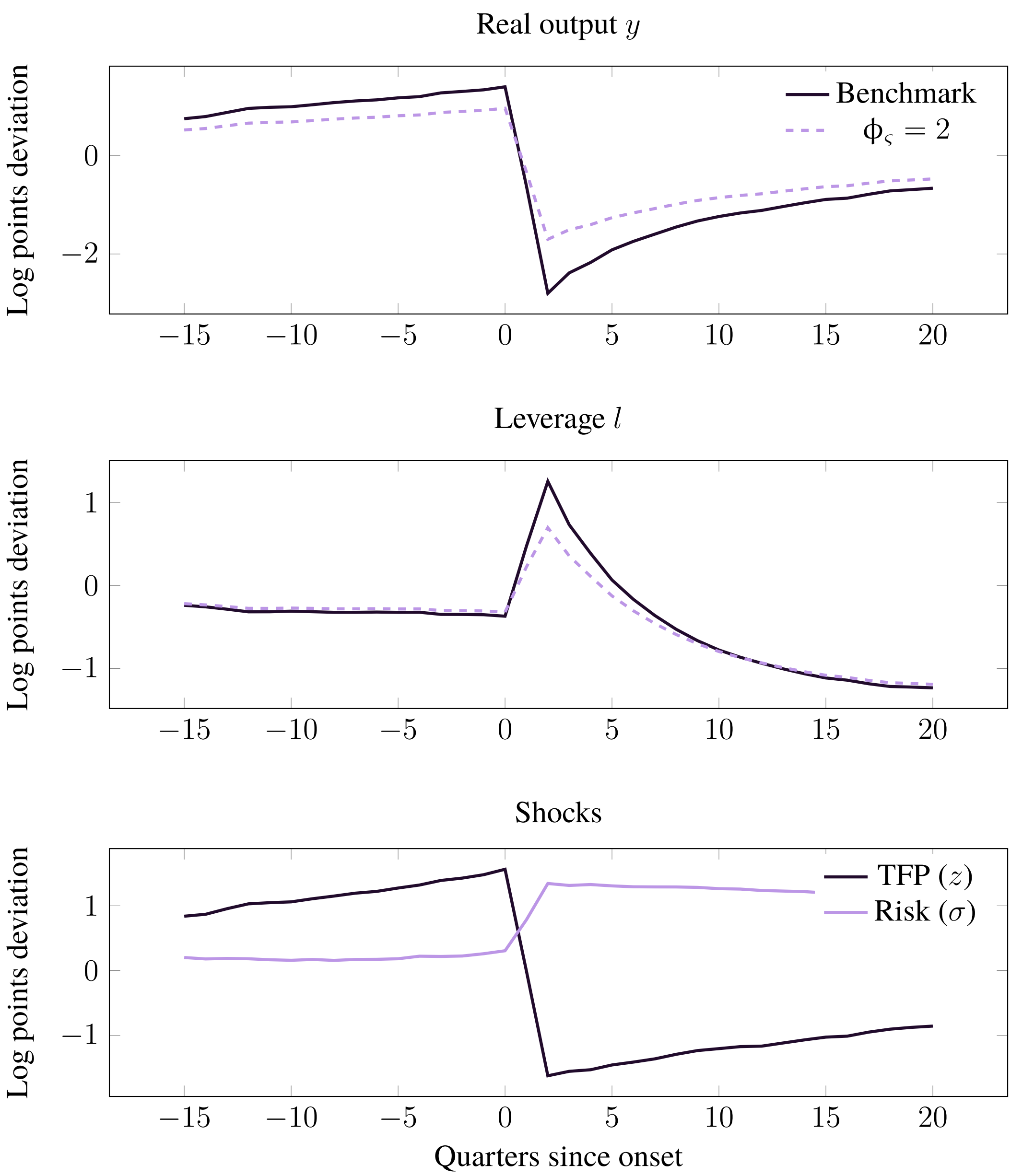
Notes: Each dot represents a sample drawn from the posterior of the estimated model. Given the associated vector of parameter values, the optimal wage subsidy simple rule is computed, and the for the optimal simple rule, the coefficient of the wage subsidy simple rule on output, and the associated welfare gain, are reported. The welfare gain is reported as the reduction in the welfare costs of business cycles as a percentage of those welfare costs in the absence of policy intervention.

estimates and generate model simulated data. From the simulated data, we extract episodes where real GDP falls by more than 4 percentage points over two quarters, reflecting the largest two quarter fall in de-trended real GDP during the US Great Recession.

The first panel of Figure 6 shows real output across the benchmark (i.e., no wage subsidy) and wage subsidy simple rule regimes. Under the wage subsidy simple rule regime, the typical depth of crises is smaller. When the coefficient of the simple rule on output is $\phi_\varsigma = 2$, the mean peak-to-trough fall in output is 28 percent smaller than in the no-policy counterfactual.

So, the wage subsidy simple rule would reduce the amplitude of business cycles in a frictionless economy, but to increase welfare in our model, the policy must reduce the cost of the frictions. The second panel shows the path of leverage under the two regimes. Despite increasing output relative to the benchmark in crisis episodes, the wage subsidy policies decrease (relative) leverage, and hence the resulting cost of financial frictions in crisis episodes. Leverage in our model is risk-based, and de-

Figure 6: Economic crises with and without wage subsidy simple rules



Notes: All values are reported as mean log deviations from deterministic steady-state levels. Solid series represent the benchmark (no wage subsidy) regime, while dashed series represent an economy with a wage subsidy simple rule with a coefficient on output of 2. Crises are episodes where real output falls by at least 4 percentage points over two quarters in simulated data generated using parameters drawn from the model posterior. For each parameter vector, we then impose a common wage subsidy simple rule and generate new simulated data using the same shock sequences to generate the dashed series, extracting periods that were identified as crises under the benchmark regime.

fined as the ratio of output to the opportunity cost of net worth. For wage subsidies to reduce leverage and increase output, they must have a precautionary impact on firms' risk taking in advance of the crisis episodes. This is the mechanism through which wage subsidy policies not only reduce the amplitude of business cycles, but also increase welfare in the model.

Crises also illustrate the limitations of the simple-rule approach to wage subsidy policies. In the absence of wage subsidies, the labour wedge increases by 4.47 percentage points as the economy moves from peak to trough. With the wage subsidy simple rule, the labour wedge decreases by 0.87 percentage points: the endogenous factor wedge τ increases by 3.48 percentage points due to the increases in leverage and risk, but the increase in the factor wedge is offset by the wage subsidy, which reduces the total labour wedge by 4.22 percentage points.[13]

Ten quarters after the onset of the crisis, the labour wedge under the wage subsidy simple rule is 2.04 percentage points smaller than pre-crisis levels, as the wage subsidy has remained high despite the fall in leverage and the factor wedge. This is likely to be inefficient: at this point, leverage has returned to near pre-crisis levels under both policy regimes. Output remains well below trend, but low output is not inconsistent with efficient allocations given the persistent effects of the exogenous shocks. At this point, the wage subsidy resulting from the simple rule is likely to be doing more harm than good.

The third panel of Figure 6 shows the typical paths of exogenous shocks around crisis periods. Crisis episodes are typically generated by a combination large negative technology shocks, contractionary risk shocks, and below steady state leverage. Further figures describing the paths of the factor wedge and consumption risk sharing are provided in Appendix J.1.

4.2 Distorted labour markets: Dixit's critique

It is argued above that the way to interpret the wage subsidy is as a prudential fiscal stimulus; an intervention that relaxes an incentive-compatibility constraint (in finan-

[13]Given wage subsidy ς, the labour wedge becomes

$$1 - \frac{-u_h}{u_c \mathbb{E}[\theta] f'(h)} = \tau - (1-\tau)\varsigma$$

cial contracting), increases welfare and boosts economic activity. Viewed from the perspective of optimal tax theory, the subsidy then in this model is as an Arnott-Stiglitz intervention. Dixit (2003) has cautioned that such interventions are subject to two key criticisms when it comes to policy- relevant insights. First, many potential policy levers impact on the distorted margin so that designing a fully optimal policy would seem far from trivial. Second, and more important for the present paper, is that when the policy intervention is acting on an already distorted margin, the policymaker is no longer trading off first order benefits and second order costs, but instead first order benefits *and* first order costs. The wage subsidy policies that we propose act on the labour margin, which is typically highly distorted in modern economies. We add a constant 40% labour tax to the baseline model and simulate the model to find the effects of optimal policy simple rules from this new distorted base. Tax revenues are rebated lump sum to households. That 'wedge' is a little higher than the average tax wedge on the average OECD worker[14] and is in the ballpark of some of the estimates for the US over a long run of data calculated by Mulligan (2002).[15] It is important to point out that in Mulligan's data the wedge is not solely the result of taxes, a point we return to presently. We find that after introducing the 40% labour 'tax', the optimal coefficient of the wage subsidy simple rule on output, ϕ_{ς}, increases from 1.80 to 3.17. The resulting expected welfare gain increases from 20.5% to 30.3% of counterfactual business cycle welfare costs.

The invariance of optimal wage subsidy policy to static tax distortions is perhaps not surprising; the relevant margin for our wage subsidy policy is not the steady state labour margin, but instead the dynamic allocation of labour across business cycle outturns. In the following section, we introduce a time-varying labour market distortion in the form of New Keynesian product market nominal rigidities.

5 Optimal wage subsidies and time varying markups

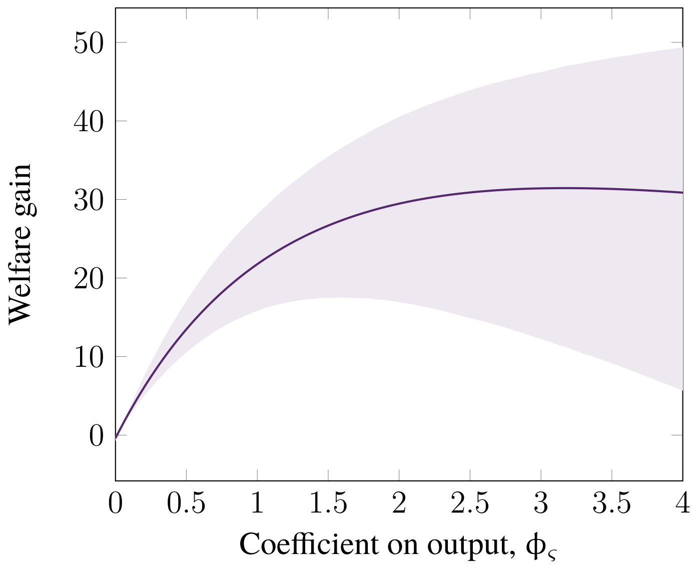
In this section we present a linearized, small scale New Keynesian version of our model. The full derivation of the model can be found in Duncan, Mainente, and

[14] See the OECD, Taxing Wages 2022.

[15] A Century of Labor-Leisure Distortions, NBER Working Paper 8774.

Figure 7: Estimated mean welfare gain from optimal simple wage subsidy rule

(Constant labour tax of 40%)



Notes: The welfare gain is reported as the reduction in the welfare costs of business cycles as a percentage of those welfare costs in the absence of policy intervention. Shaded area indicates 90% credible interval. Parameter values are estimated under the assumption that there is no wage subsidy policy.

Nolan (2023). Our primary motivation for introducing nominal rigidities is to further address Dixit's critique: nominal rigidities introduce cyclical fluctuations in retail markups. The resulting fluctuations in the labour wedge interact with our cyclical wage subsidy policy regime. If our wage subsidy regime were to amplify the costs of cyclical fluctuations in markups, then this could overturn the result that countercyclical wage subsidy policies improve welfare.

We are able to verify, however, that countercyclical wage subsidy policies remain welfare improving in the presence of time varying product market distortions that generate cyclical fluctuations in the labour wedge of inefficiency (Proposition 3). Under jointly optimal monetary and wage subsidy policies, inflation is stabilised for uncertainty shocks, and wage subsidies are phased out quickly after their introduction (Section 5.2). Moreover, in the presence of aggregate demand externalities, optimal wage subsidy policies are not as severely affected by the time inconsistency problem as in the flexible price economy (Section 5.3. The flexible price case was analysed in Section 3.1). The proofs of all propositions in this Section are contained in Appendix I.

The New Keynesian version of our model is characterised by three principle equations (15, 16, and 17).

The IS curve

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\gamma} r_t + \omega(1-\varphi)\mathbb{E}_t[\Delta l_{t+1}] - \omega\varphi\mathbb{E}_t[\Delta\sigma_{t+1}], \tag{15}$$

the Phillips curve

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \lambda \text{pp}_t, \tag{16}$$

and the Leverage curve

$$\Delta l_t = \tfrac{1}{1+\gamma\omega(1-\varphi)} \left(-\varphi(l_{t-1} + \sigma_{t-1}) + \gamma\omega\varphi\Delta\sigma_t - (\gamma-1)\Delta x_t\right), \tag{17}$$

where $\frac{\omega}{1+\omega}$ is the steady state entrepreneurial consumption share, and φ is the elasticity of the equity risk premium with respect to leverage and risk. In this Section, output y, leverage l, uncertainty σ, technology z, and producer prices pp are expressed in terms of their respective log deviations from deterministic steady state levels. The real interest rate is $r_t := i_t - \mathbb{E}_t[\pi_{t+1}]$ where nominal interest rate is i and inflation π. The factor wedge τ, and the wage subsidy ς are expressed in terms of their respective deviations from deterministic steady state levels. The operator Δ takes the growth rate of its argument, $\Delta l_t = l_t - l_{t-1}$.

Our IS curve differs from the benchmark New Keynesian IS curve as a result of the time varying distribution of consumption between entrepreneurs and the representative household.[16] Our IS curve is derived from the representative household's optimal consumption savings plan. Expected household consumption growth depends on expected output growth but also current and expected future leverage and uncertainty: an increase in leverage or uncertainty reduces the expected growth of the households' consumption share of total output.

Retailers purchase the wholesale good from entrepreneurs in competitive markets. From this wholesale good the retailers produce a differentiated retail good. The retail goods are sold in monopolistically competitive markets subject to Calvo pricing frictions. The producer prices paid by the retailers for the wholesale good are equal to the total marginal costs of entrepreneurial production. These costs comprise marginal labour costs and the risk premium that constitutes the labour wedge of inefficiency τ. Marginal labour costs are the sum of traditional New Keynesian marginal costs (increasing in the output gap), a wealth effect on labour supply resulting from fluctuations in the distribution of consumption (when the household's consumption share is high, wages demanded are higher for every level of the output gap), and the wage subsidy policy instrument ς.

$$\mathrm{pp}_t = \underbrace{\left(\gamma + \tfrac{\varphi+\alpha}{1-\alpha}\right) x_t - \tfrac{1+\varphi}{1-\alpha}\, z_t}_{\text{benchmark model marginal costs}} + \underbrace{\gamma\omega(1-\varphi)l_t - \gamma\omega\varphi\sigma_t}_{\text{consumption inequality wealth effect}} + \underbrace{\tau_t}_{\text{labour wedge}} - \underbrace{\varsigma_t}_{\text{wage subsidy}} . \tag{18}$$

The Leverage curve (17) is unchanged from our benchmark flexible price model up to log-linearisation. The policymaker's welfare function (provided in Appendix I) differs from the flexible price welfare function as a result of the social costs of price dispersion during periods of abnormally high or low inflation.

[16]For an exposition of the benchmark New Keynesian model, refer to Galí (2008, Chapter 3).

5.1 Optimal wage subsidy policy taking the monetary policy rule as given

In this subsection we close the model with a Taylor-type monetary policy rule that responds to current inflation. We also impose log-utility for households to simplify exposition.

Assumption 1 *Worker households have log utility, $\gamma = 1$. Interest rates follow the rule: $i = \phi_\pi \pi_t$*

Proposition 3 *Given Assumption 1,*

(i) optimal wage subsidies increase in response to cost-push components of fluctuations in leverage and uncertainty, increase in response to contractionary supply shocks, and increase in response to expansionary aggregate demand components of fluctuations in leverage and uncertainty.

(ii) Optimal Wage subsidies are temporary, and are withdrawn more quickly than the underlying shocks dissipate.

(iii) The optimal wage subsidy policy is characterised by Equation 20.

Combining the equilibrium conditions yields the following characterisation of optimal policy. First, the optimal path of inflation follows

$$\pi_t = \mu \pi_{t-1} + s_t \tag{19}$$

$$\varsigma_t = -\mu \left(\lambda \chi \frac{\phi_\pi - \mu}{1 - \mu} + (1 - \beta \mu) \right) \pi_{t-1} + u_t \tag{20}$$

where s_t, u_t are functions of current period shocks and leverage (which is independent of policy under log utility), increasing in the cost-push and aggregate demand components of those shocks. The endogeneous persistence of inflation and output under optimal wage subsidy policy is captured by μ, a real-valued composite parameter in $(0, 1)$. Full characterisations of s_t, u_t, μ are provided in the Appendix I.

Leverage and risk both increase marginal costs of production in our model, similar to New Keynesian cost-push shocks. Following a standard New Keynesian cost-push shock, an optimal wage subsidy policy would completely offset the contribution of the shock to marginal costs, restoring the first best allocation. The optimal wage subsidy policy would then be withdrawn as the shock dissipates. In contrast with standard New Keynesian cost-push shocks, wage subsidies in our model cannot restore first best efficient allocations in response to uncertainty and technology shocks in our model. Welfare costs of uncertainty shocks and leverage dynamics remain even when the paths of output, real interest rates and inflation are stabilised. Optimal wage subsidies are withdrawn more quickly than the underlying shocks dissipate, as their primary role is prudential, and is most impactful in response to the unpredictable component of the shock on impact, rather than the predictible persistent component of the shock over time.

Why is this prudential? Start with the leverage updating relationship (17), adjusted for log utility:

$$\Delta l_t = \tfrac{1}{1+\omega(1-\varphi)}\left(-\varphi(l_{t-1}+\sigma_{t-1}) + \omega\varphi\Delta\sigma_t\right) \tag{21}$$

It is apparent that leverage is invariant to current period output and current period wage subsidies. Wage subsidies increase output, so for leverage to be invariant to wage subsidies, it must be the case that entrepreneurs bring more wealth into periods where wages are subsidised; conditional wage subsidies must generate an ex ante portfolio reallocation. If wage subsidies are introduced during downturns, then wage subsidies will motivate precautionary behaviour, where entrepreneurs bring more wealth into downturns.

One can see what happens when this precautionary channel is closed off. This is revealed in the leverage updating rule when only trade in non-contingent nominal bonds is permitted and aggregate risk markets are shut down. The resulting leverage updating relationship is

$$\Delta l_t = -\varphi l_{t-1} - \varphi\sigma_{t-1} - (i_{t-1} - \pi_t) + \Delta y_t \tag{22}$$

Comparing 22 with 21 shows the importance of the prudential effect of wage subsidy policies. Under closed aggregate risk markets (22), an increase in current period output (relative to current period inflation) increases current period leverage, increasing current period marginal costs and reducing the effectiveness of the wage-subsidy policy.

5.2 Joint optimal wage subsidy and monetary policy

Under log utility, optimal wage subsidy and monetary policies eliminate fluctuations in output and producer prices resulting from uncertainty shocks, similar to the optimal response to markup shocks in the standard New Keynesian model.

When households are more risk averse, $(\gamma > 1)$ optimal policy no longer eliminates fluctuations in output resulting from uncertainty shocks.

Remark 1 *Under joint optimal monetary and wage subsidy policy, the optimal path of inflation is zero in all periods* $\pi_t = 0\ \forall t$.

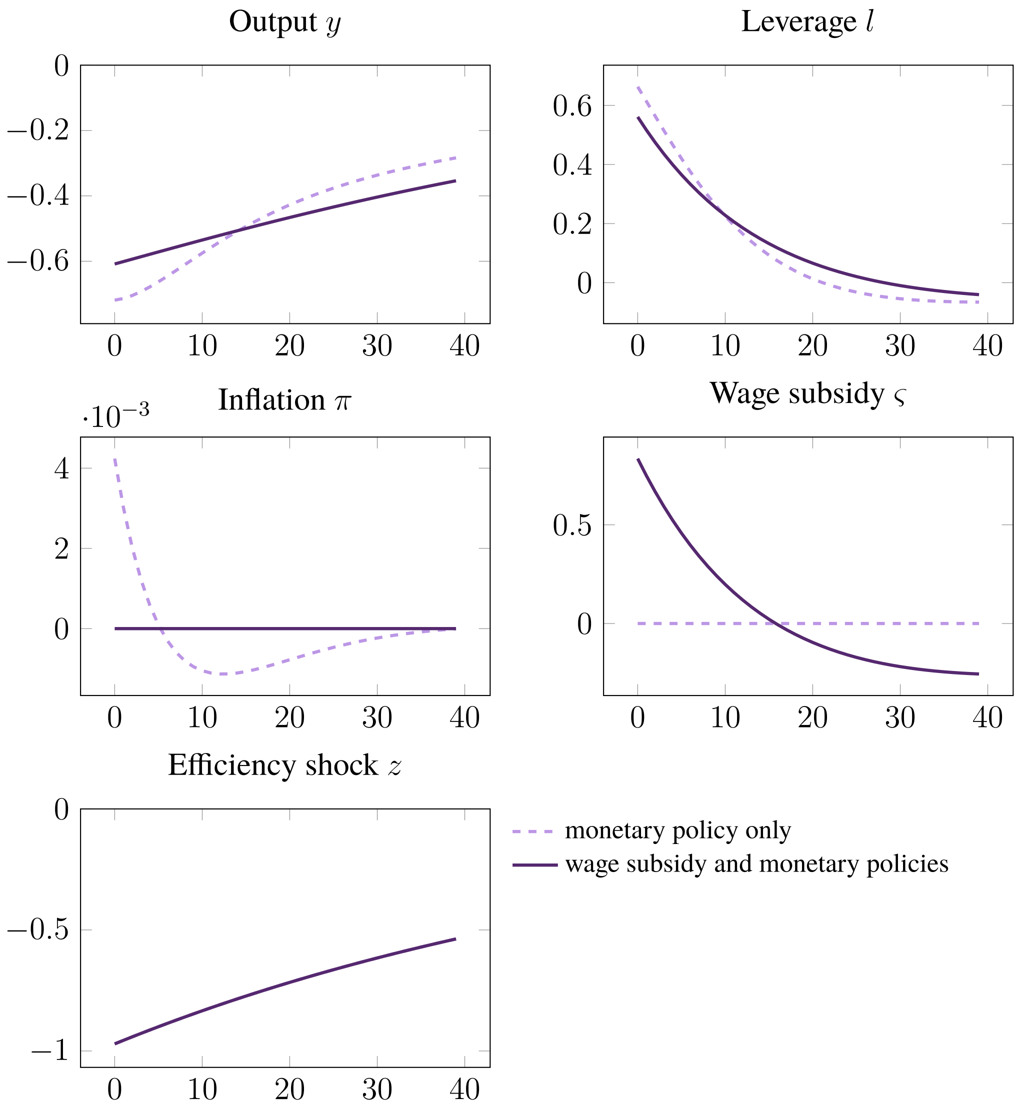
The proofs of Remark 1 can be found in Appendix I. It is straightforward to show that Remark 1 also holds in response to traditional New Keynesian markup shocks when the policymaker has access to wage subsidies.[17] The convexity of monitoring costs generates a wedge between the social marginal costs of leverage and the private marginal costs of leverage. Intuitively, when the convexity of monitoring costs is high, fluctuations in leverage have large social costs, and optimal wage subsidy policy seeks to smooth the path of leverage. By inspection of 17, we can see that the path of leverage is smoother if output falls when leverage is high.

Figure 8 presents the economy's response to a recessionary efficiency shock in the model under optimal monetary policy and under optimal combined monetary and wage subsidy policy. Under jointly optimal policy, wage subsidies increase sharply in response to the shock, damping the output, leverage, and inflation responses to the shock. Optimal wage subsidies decay more quickly than output or the efficiency shock itself.

[17] Another similarity with markup shocks is that in the absence of wage subsidies, the optimal monetary policy allows inflation to deviate from target in response to uncertainty shocks.

Figure 8: Optimal monetary and wage subsidy policies under commitment

Optimal monetary policy and jointly optimal monetary and wage subsidy policy responses to recessionary 1sd efficiency shock.



Values are expressed in log point deviations from steady state. The estimated model is closed with the Taylor-type interest rate rule $i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_y(y_t - \varrho z_t)\right) + \varepsilon_{it}$, where $\varrho := \frac{\chi}{\chi+\gamma-1}$. Model estimation details can be found in Appendix G.

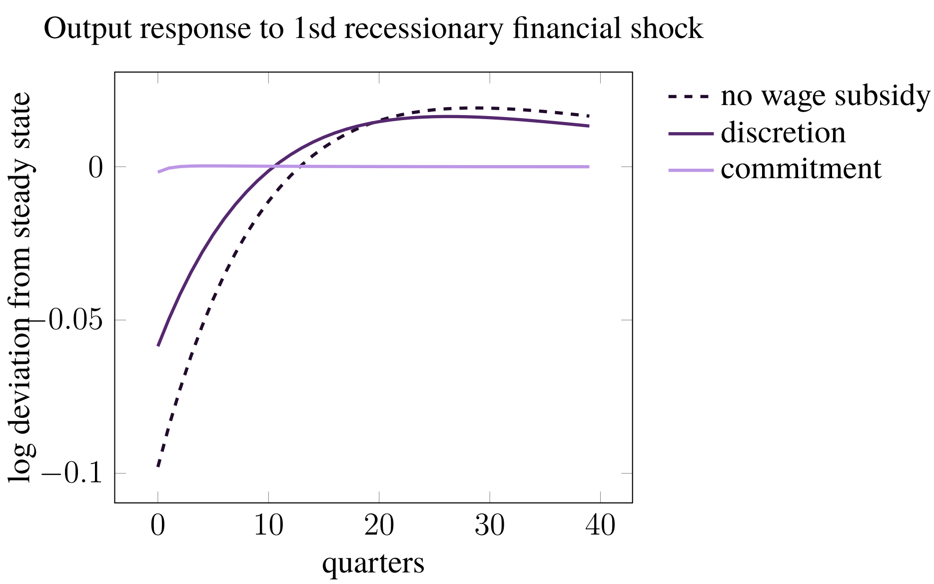
5.3 Commitment versus discretion under log utility ($\gamma = 1$)

In the flexible price model, we saw that a fiscal policymaker optimising under discretion would not levy countercyclical wage subsidies (Section 3.1, Proposition 2). Ex post, any wage subsidy in the flexible price model introduces a distortion into the labour market, reducing welfare. The welfare gains from countercyclical wage subsidies in the flexible price model result from the precautionary behaviour of entrepreneurs who anticipate the introduction of future wage subsidies in downturns. Conditional upon that precautionary behaviour ex ante from entrepreneurs, the policymaker has no incentive to introduce the wage subsidy when the downturn arrives. In equilibrium, there is no precautionary behaviour, and there are no wage subsidies. In the presence of nominal rigidities, this laissez-faire result is overturned. Aggregate demand externalities motivate a policymaker under discretion to stimulate the economy in downturns, including with their wage subsidy instrument. Anticipating the discretionary policymaker's incentive to stimulate the economy with wage subsidies, entrepreneurs will exercise ex ante precaution, generating a prudential benefit.

In the presence of aggregate demand externalities, optimal wage subsidies under discretion still respond to shocks. High leverage and uncertainty generate cost pressure through the Phillips curve, and optimal wage subsidies under discretion respond to this cost pressure. Figure 9 presents an example showing the response of output to a recessionary uncertainty shock in the absence of wage subsidy policy, under a discretion regime and a commitment regime. Under commitment, the optimal policy eliminates most of the volatility of output, and hours, in response to the shock, a small departure from the flexible price model with log utility where output is completely stabilised under commitment in response to uncertainty shocks (Proposition 1.b).[18] Under discretion, the optimal policy still dampens the volatility of output, in contrast with the flexible price case. The optimality of countercyclical wage subsidy policy under discretion is largely due to the coincidence of aggregate demand externalities and the leverage externality. The wage subsidy policymaker's efforts to reduce the cost-push consequences of high leverage have an unintended

[18] This should not be confused with restoring first best efficient allocations—the distribution of consumption remains volatile, and the entrepreneur still bears welfare costs of higher idiosyncratic consumption risk.

Figure 9: Optimal wage subsidy policies under commitment and discretion



Notes: Values are expressed in log point deviations from steady state. The model is re-estimated to produce this plot with two additional parameter restrictions: log household utility ($\gamma = 1$) and the simplified Taylor-type interest rate rule $i_t = \phi_\pi \pi_t$.

prudential effect. In equilibrium, discretionary countercyclical wage subsidies motivate a precautionary response from firms ex ante, who bring more equity into the recession than in the absence of the wage subsidies.

6 Discussion

When governments and policymakers intervene in the economy to affect aggregate outcomes or support particular sectors or agents, it may be in crisis conditions when thoughts of moral hazard are set to one side. As Goodhart noted:

> The time to worry about moral hazard is in the boom. The first priority is to get out of the present hole. Worrying about moral hazard in [crisis] circumstances is rather like refusing to sell fire insurance just after the Great Fire of London for fear of adversely affecting future behaviour.

[19]

However, as we now know all too clearly, financial crises are not the only crises which have resulted in policy intervention on a substantial scale. Future pandemics, the effects of climate change, wars and other massive shocks to the economy (e.g., natural disasters), may all result in calls for policymakers to act on a grand scale. Consequently, there is a need to consider the design of policies in such a way that, if necessary, they can be implemented effectively in crisis conditions while mitigating and not promoting moral hazard, adverse selection and other distortions. In this paper, we add to the contributions of Milne (2020), Romer and Romer (2022), Woodford (2022), Guerrieri et al. (2022) and others. These papers study the roles of social insurance and stimulus in addressing crises whilst we shift the focus to the concern alluded to, but unaddressed, by Goodhart's observation. That is, we have begun the investigation into the design of policies that provide stimulus to a depressed economy whilst avoiding damaging the underlying incentive of firms to behave prudently.

Our models are quite simple and so one must be cautious about generalizing too readily our findings. Or, to put it differently, our findings are suggestive and need to be verified in richer environments. Nevertheless, we think several indicative insights stand out.

First, it is in principle possible to design prudential stimulus programs, given they are rationally anticipated.[20] Following the insights of Arnott and Stiglitz (1986) the idea is to design policies which tax the complements of moral hazard and subsidise the substitutes. Sometimes critics of fiscal stimulus point to 'lags' in the implementation of particular projects meaning they come too late to be of help in a crisis, or even to the difficulty of identifying 'shovel-ready' projects. Our first insight suggests the challenges in identifying desirable stimulus policies also need to pass a 'prudential' test.

Second, once identified, prudential stimulus policies, again in principle, need

[19]Not the Time to Worry about Moral Hazard by Charles Goodhart, *Financial Times*, September 18 2008.

[20]That is, we modeled wage subsidy schemes that were implemented with no uncertainty. That may not be a harmless assumption. On the one hand, if there were some uncertainty as to whether such a scheme was to be implemented that may detract from the schemes effectiveness. On the other hand, as we observe in the text, most schemes were already in place when the Covid pandemic hit.

not fall foul of one aspect of Dixit's (2003) critique that that pre-existing distortions might scupper the chances of finding a successful policy: the positive welfare effects of our prudential wage subsidy stimulus policy are robust to the presence of steady state and dynamic labour wedges in the form of labour income taxes and retail markups. Third, the scheme should likely build on pre-existing interventions. In the above model, there is a steady-state friction due to the underlying asymmetric information problem. It follows that there should be a steady-state policy intervention. That leads to: Fourth, the policy is likely to be dynamic, perhaps responding strongly as soon as the shock hits, but being withdrawn as time passes, as it did above with the wage subsidy. Fifth, broad-based prudential fiscal stimulus can be designed to "get in all the cracks." During the Covid-19 pandemic, many countries enacted broad-based policies that acted in part like wage subsidies and that were designed to affect many sectors of the economy. Unlike traditional macro-prudential policy tools, wage subsidies do not require transmission through the banking sector. This is particularly important if transmission of macro-prudential policy through the banking sector is under strain either as a symptom or cause of the underlying recession.

In addition to investigating the robustness of these indicative findings, there are several other issues that are important to pursue. First, we have focused our attention in this paper on prudential stimulus, namely fiscal subsidies enacted in downturns that promote the ex ante prudence of firms. But we could also attain welfare gains by taxing the complements of moral hazard: our approach can be extended to the design of prudential austerity programmes. Prudential austerity programmes, when anticipated, could reduce the likelihood and severity of recessions, ultimately reducing the social costs of the recession itself and the resulting austerity. Second, Dixit's other critique of Arnott-Stiglitz interventions is that they are not likely to be unique. It may turn out, for instance, in designing prudential stimulus packages that it is less distortionary to subsidise investment rather than subsidise labour. Or it may be that a combination of investment tax credits and wage subsidies is preferable (as for example may be the case if smoothing the intervention across a number of ex ante distorted margins). That conclusion would depend, amongst other things on an assessment of the distortions on hiring labour relative to the frictions in the market for outside finance, the tax distortions affecting investment, and so on, as well as the welfare cost of raising tax revenue. It may be the case that the design of the optimal stimulus package differs across sectors of the economy. Third, adopting a prudential perspective to the design of fiscal policy involves a fairly fundamental change of perspective that would doubtless be politically as well as technically challenging. So, with the right policies, the choice between ex ante prudential regulation and cleaning up after the mess ex post may not be as stark as hitherto thought. However, enabling government intervention, perhaps on a large scale, to induce increased private sector efficiency is a desirable feature of any fiscal plan.

References

Richard Arnott and Joseph E. Stiglitz. Moral hazard and optimal commodity taxation. *Journal of Public Economics*, 29(1):1–24, February 1986.

Robert J. Barro and David B. Gordon. Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics*, 12(1):101–121, 1983. ISSN 0304-3932. doi: https://doi.org/10.1016/0304-3932(83)90051-X.

Ben S. Bernanke, Mark L. Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1, Part C, chapter 21, pages 1341–1393. Elsevier, 1 edition, 1999.

Kim C Border and Joel Sobel. Samurai accountant: A theory of auditing and plunder. *Review of Economic Studies*, 54(4):525–40, October 1987.

Charles T. Carlstrom, Timothy S. Fuerst, and Matthius Paustian. Optimal Contracts, Aggregate Risk, and the Financial Accelerator. Working Paper 1420, Federal Reserve Bank of Cleveland, October 2014.

Sebastian Di Tella. Uncertainty shocks and balance sheet recessions. *Journal of Political Economy*, 125(6):2038–2081, 2017. doi: 10.1086/694290.

Avinash Dixit. Whither Greenwald-Stiglitz?, October 2003. Presentation at Joseph Stiglitz's 60th birthday conference, Columbia University.

Alfred Duncan and Charles Nolan. Disputes, debt and equity. *Theoretical Economics*, 14(3), September 2019.

Alfred Duncan and Charles Nolan. Destabilizing insurance, 2021.

Alfred Duncan, João Pedro De Camargo Mainente, and Charles Nolan. Optimal Macro-Financial Policies in a New Keynesian Model with Privately Optimal Risk Taking, 2023.

International Monetary Fund. Policy tracker, 2021. URL https://www.imf.org/en/Topics/imf-and-covid19/Policy-Responses-to-COVID-19. Accessed: 2021-02-09.

John Kenneth Galbraith. *Money, Whence it Came, where it Went*. Pelican books. Houghton Mifflin, 1975. ISBN 9780395198438.

Jordi Galí. *Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Introductory Chapters. Princeton University Press, 2008.

Giulia Giupponi, Camille Landais, and Alice Lapeyre. Should we insure workers or jobs during recessions? *Journal of Economic Perspectives*, 36(2):29–54, May 2022. doi: 10.1257/jep.36.2.29.

Bruce C Greenwald and Joseph E Stiglitz. Externalities in Economies with Imperfect Information and Incomplete Markets. *The Quarterly Journal of Economics*, 101(2):229–64, May 1986.

Veronica Guerrieri, Guido Lorenzoni, Ludwig Straub, and Iván Werning. Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages? *American Economic Review*, 112(5):1437–1474, May 2022. doi: 10.1257/aer.20201063.

Nicholas Kaldor. Wage subsidies as a remedy for unemployment. *Journal of Political Economy*, 44(6):721–742, 1936. ISSN 00223808, 1537534X.

Weerachart T. Kilenthong and Robert M. Townsend. A Market Based Solution to Price Externalities: A Generalized Framework. NBER Working Papers 20275, National Bureau of Economic Research, Inc, July 2014.

Mervyn King. *The End of Alchemy: Money, Banking and the Future of the Global Economy*. Little, Brown, 2016. ISBN 978-0-349-14067-4.

Nobuhiro Kiyotaki and John Moore. Credit Cycles. *Journal of Political Economy*, 105(2):211–48, April 1997.

Arvind Krishnamurthy. Collateral constraints and the amplification mechanism. *Journal of Economic Theory*, 111(2):277–292, 2003.

Finn E. Kydland and Edward C. Prescott. Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, 85(3):473–491, 1977. ISSN 00223808, 1537534X.

Alistair Milne. A critical COVID-19 economic policy tool: Retrospective insurance, March 2020. URL `https://dx.doi.org/10.2139/ssrn.3558667`.

Dilip Mookherjee and Ivan Png. Optimal auditing, insurance, and redistribution. *The Quarterly Journal of Economics*, 104(2):399–415, May 1989.

Takashi Negishi. Welfare economics and existence of an equilibrium for a competitive economy. *Metroeconomica*, 12(2-3):92–97, 1960. doi: 10.1111/j.1467-999X.1960.tb00275.x.

Kalin Nikolov. Collateral amplification under complete markets. *Journal of Economic Dynamics and Control*, 45(C):80–93, 2014.

International Labour Organization. Temporary wage subsidies: Country examples, an appendix to the ilo factsheet: Temporary wage subsidies, May 2020.

Edmund Phelps. *Rewarding Work*. Harvard University Press, 1997. ISBN 9780674026940.

Arthur Pigou. *The Theory of Unemployment*. Macmillan, 1933.

Edward Prescott and Robert Townsend. Pareto optima and competitive equilibria with adverse selection and moral hazard. *Econometrica*, 52(1):21–45, 1984.

Christina D. Romer and David H. Romer. A social insurance perspective on pandemic fiscal policy: Implications for unemployment insurance and hazard pay. *The Journal of Economic Perspectives*, 36(2):3–28, 2022. ISSN 08953309, 19447965.

Robert M. Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293, October 1979.

Michael Woodford. Effective Demand Failures and the Limits of Monetary Stabilization Policy. *American Economic Review*, 112(5):1475–1521, May 2022. doi: 10.1257/aer.20201529.

A Aggregate risk sharing in the competitive equilibrium

To derive the aggregate risk sharing condition in the main text we first set out the optimal savings decision of the two sets of agents, entrepreneurs (denoted with superscript e) and households. There is a unit measure of each agent. The aggregate state is denoted s and θ denotes the entrepreneur's idiosyncratic state. Agents in the model can buy and sell aggregate-state (Arrow-Debreu) risk assets as well as invest in risky, idiosyncratic and risk-free assets. Thus, one obtains for individual entrepreneur j:

$$\beta^e E_\theta u^{e\prime}(c^e_{j,t+1}) = u^{e\prime}(c^e_{j,t})p_t(s_{t+1})$$
$$\beta u'(c_{t+1}) = u'(c_t)p_t(s_{t+1}).$$

With a slight abuse of notation, E_θ denotes solely the expectation over the realisation of idiosyncratic states, in order to be explicit about the sources of uncertainty. Lagging by one period the foregoing expressions, since time t idiosyncratic risk is intra-period and is not resolved (unlike s_t) before other time t decisions have to be made (like hiring labour, investing in risky idiosyncratic production), it follows from rearrangement that

$$\beta^e \left(\frac{E_\theta u^{e\prime}(c^e_{j,t})}{u^{e\prime}(c^e_{j,t-1})} \right) = \beta \left(\frac{u'(c_t)}{u'(c_{t-1})} \right).$$

Within period t, at the time of negotiating external finance, non-contingent savings and loans are the opportunity cost of state-contingent loans, therefore

$$(1 + r_{t+1})E_\theta u^{e\prime}(c^e_{j,t+1}) = E_\theta u^{e\prime}(c^e_{j,t+1})R_{t+1}(\theta, s_{t+1}).$$

Note that given intertemporal homotheticity and scalable technologies, the function $R_{t+1}(\theta, s_{t+1})$ is independent of the past history of individual entrepreneur j. So, we re-write the risk sharing relationship above as

$$\beta^e \left(\frac{E_\theta u^{e\prime}(c^e_{j,t+1})R_{t+1}(\theta, s_{t+1})}{u^{e\prime}(c^e_{j,t-1})} \right) = \beta(1 + r_{t+1}) \left(\frac{u'(c_t)}{u'(c_{t-1})} \right).$$

We want to express the above risk sharing relationship in terms of aggregate levels of entrepreneurial consumption, rather than the paths of individual entrepreneurs' consumption. Consider the product $E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)]$.

Under log utility

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = E_\theta\left[\frac{R_t(\theta, s_t)}{c^e_{j,t}}\right].$$

Consumption decisions in period t are made after the individual realisation of R_t, and at the time of consumption, the total wealth available to individual entrepreneur is proportionate to their realisation of R_t. It follows, given intertemporal homotheticity, that $c^e_{j,t} = kR_t(\theta, s_t)$ for some $k \perp \theta$. The term k is not necessarily constant, but is independent to the time t realisation of θ. It follows that

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = \frac{1}{k}.$$

Now, let $\bar{c}^e_{j,t} = E_\theta[c^e_{j,t}]$, $\bar{R}_t = E_\theta[R_t(\theta, s_t)]$. By construction of k,

$$\bar{c}^e_{j,t} = k\bar{R}_t.$$

Therefore we have

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = \frac{\bar{R}_t}{\bar{c}^e_{j,t}}.$$

Under log utility,

$$E_\theta[u^{e\prime}(c^e_{j,t})R_t(\theta, s_t)] = u^{e\prime}(\bar{c}^e_{j,t})\bar{R}_t.$$

To summarise, under log utility, the expected product of marginal utility and gross returns to entrepreneurial wealth is equal to the product of the marginal utility of expected consumption and the expected gross return to entrepreneurial wealth.

Now, we set the index j to denote the entrepreneur who exits period $t-1$ with consumption equal to the mean consumption across all entrepreneurs, which we denote $\bar{c}^e_{t-1}$. It follows that

$$\beta^e\left(\frac{u^{e\prime}(\bar{c}^e_t)}{u^{e\prime}(\bar{c}^e_{t-1})}\right)E_\theta\left[\frac{R_t(\theta, s_t)}{1+r_t}\right] = \left(\frac{u'(c_t)}{u'(c_{t-1})}\right).$$

Re-writing the previous expression with $\rho \equiv \frac{E_\theta R(\theta,s)}{(1+r)}$, $\lambda \equiv u'(c)/u^{e\prime}(\bar{c}^e)$,

$$\lambda_t = \lambda_{t-1}\frac{\beta^e}{\beta}\rho_t,$$

where λ_t is measurable in the time t information set. This is the aggregate risk-sharing result of the main text.

Under perfect risk sharing for individual specific risks, the equity risk premium would disappear ($\rho = 1$), and we would be left with the traditional risk sharing relationship $\lambda_t = \lambda_{t-1}\frac{\beta^e}{\beta}$. The equity risk premium reflects the wedge between entrepreneur's expected marginal utility of consumption and the marginal utility of expected entrepreneurial consumption. An important consequence of imperfect idiosyncratic risk sharing is that it might not be possible to test aggregate risk sharing from aggregate consumption data alone.

B The entrepreneur's problem

$$v^e(q^e) = \max_{x^e, c^e, q^e, h, r(\theta,\vartheta)} \mathbb{E}_{\Theta,S}\left\{u^e(c^e) + \beta^e v^e(q^{e\prime})\right\}$$

subject

$$Qq^{e\prime} = R(\theta,s)q^e - c^e + y + Q(1-\delta)q^e - qr(\theta,\vartheta) - wh - \int_{s\in S} p(s)x^e(s)ds + x^e(s')$$

$$y = \theta z f(K,h)$$

$$K = q^e + q$$

$$r(\underline{\theta},\overline{\vartheta}) \geq r(\overline{\theta},\emptyset) \tag{B1}$$

$$\sum_{(\theta,\vartheta)} P(\theta,\vartheta)r(\theta,\vartheta)q \geq rq + \underline{\pi}\kappa K. \tag{B2}$$

The notation is as follows. Superscript e denotes the entrepreneur, and $v^e(q^e)$ is the value function. c^e is consumption. A prime denotes a next period variable. $R(\theta,s)$ is the return to entrepreneurial wealth, q^e, and is partly the outcome of a privately optimal external finance contract, determined at the beginning of the period, and conditional on idiosyncratic states realised within the period. θ denotes the idiosyncratic state which is private information where $\theta \in \{\underline{\theta},\overline{\theta}\}$ where $\underline{\theta} < \overline{\theta}$. These two states occur with probabilities $\underline{\pi}$ and $\overline{\pi}$ and $\underline{\pi}\underline{\theta}_t + \overline{\pi}\overline{\theta}_t = 1$.

Trade in aggregate risk markets is captured by the quantities $x^e(s)$, denoting the amount purchased of an asset with payoff 1 conditional upon the future state of the world being realised as state s. The current period price of this security is denoted $p(s)$. As indicated earlier, trade in securities indexed by the aggregate state are not hampered by any problem of asymmetric information; unlike idiosyncratic states, aggregate states are costlessly observed and verified by all agents. These markets are active.

The first constraint is the entrepreneur's budget constraint and the second is the productions function or technology constraint. The third equation defines K as the sum of the entrepreneur's and the household's wealth. The fourth equation, Equation B1, is the incentive compatibility constraint. And the final equation, Equation Equation B2 is the household participation constraint. Note that capital rental payments $r(\theta, \vartheta)$ are contingent on the idiosyncratic shock θ as well as any audit signal obtained by the household (or bank), $\vartheta \in \{\underline{\vartheta}, \overline{\vartheta}\}$. Audit signals are distributed as follows: $P(\overline{\vartheta}|\overline{\theta}) = 1, P(\overline{\vartheta}|\underline{\theta}) = \eta, P(\underline{\vartheta}|\underline{\theta}) = 1-\eta$ so that high types are always correctly identified by the audit, but with probability η low types are incorrectly tagged as high types. Thus to access outside investment, entrepreneurs need to commit to overturned low reports paying out more than high reports; that is toinduce truth-telling and is what the incentive compatability constraints ensures. To ensure the investment of households, expected loan repayments must exceed the sum of the households' opportunity cost of funds, rq plus expected audit costs, $\underline{\pi}\kappa K$ where $\underline{\pi}$ is the frequency of audit costs (and the probability of low idiosyncratic probability draws) and κK is the cost of each audit, where κ is constant. It turns out that that when audit costs are not too high and auditing is imperfect, defaultable debt contracts with deterministic audit strategies are constrained efficient; see Duncan and Nolan (2019).

C Policymaker's problem (Proof of Proposition 1)

In order to solve for the optimal wage subsidy, we present a minimal set of equilibrium conditions from the competitive economy, excluding the household's labour supply condition.

We solve for the policymaker's optimal labour supply, and use this solution to derive the wage subsidy that supports this labour supply. While non-zero wage subsidies may require lump sum transfers to finance, these transfers are ultimately traded away in competitive markets for aggregate risk, and do not appear in our policymaker's programme.

Equilibrium conditions

The following equilibrium and market clearing conditions are constraints faced by the policymaker.

$$zh^{\alpha} \geq c + c^e \tag{Λ_1}$$

$$lc^e = zh^{\alpha}(1-\beta^e)\mathcal{R}(l,\sigma) \tag{Λ_2}$$

$$\lambda' = c^{-\gamma}c^e \tag{Λ_3}$$

$$\lambda' = \frac{\beta^e}{\beta}\mathcal{R}(l,\sigma)\lambda \tag{Λ_4}$$

where $\mathcal{R}(l,\sigma) := \rho \ (= 1 + l\tau)$, where τ is a function of l, σ.
The competitive equilibrium labour supply can be expressed as follows, and is left out of the policymaker's problem:

$$h^{1-\alpha+\psi}c^{\gamma} = \alpha z(1-\tau) \tag{CE labour supply}$$

The policymaker's objective function can be expressed as follows:

$$V = \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\mu\beta^t \left(\frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\psi}}{1+\psi} \right) + \mu^e(\beta^e)^t \left(\log c_t^e + \frac{\log g(l_t,\sigma_t)}{1-\beta^e} \right) \right] \tag{C1}$$

The policymaker maximises (C3) subject to the constraints tagged by their respective Lagrange multiplers $(\Lambda_1, ..., \Lambda_4)$. The policymaker's programme can be convexified by log transformations when $\mathcal{R}, g$ are log-convex in l. Alternatively, constraint qualifications are straightforward to verify for more general specifications.

Envelope Condition

$$V_\lambda = \frac{\beta^e}{\beta}\mathcal{R}(l,\sigma)\Lambda_4$$

FONCs

$$\begin{aligned}
h: &\qquad \mu h^{1-\alpha+\psi} = \alpha z\left[\Lambda_2(1-\beta^e)\mathcal{R}(l,\sigma) - \Lambda_1\right] \\
c: &\qquad \mu = c^\gamma\left(\Lambda_3\frac{\gamma}{c}c^{-\gamma}c^e - \Lambda_1\right) \\
c^e: &\qquad 0 = \mu^e + \Lambda_1 c^e - \Lambda_2 l c^e + \Lambda_3 c^{-\gamma}c^e \\
l: &\qquad 0 = \frac{\mu^e}{1-\beta^e}\frac{g_l(l,\sigma)}{g(l,\sigma)} + \Lambda_2\left[zh^\alpha(1-\beta^e)\mathcal{R}_l(l,\sigma) - c^e\right] + \Lambda_4\frac{\beta^e}{\beta}\mathcal{R}_l(l,\sigma)\lambda \\
\lambda': &\qquad 0 = -\Lambda_3 - \Lambda_4 + \beta\mathbb{E}\hat{V}_\lambda
\end{aligned}$$

where $\hat{V}$ is a placeholder.

By constraint 3

$$\begin{aligned}
c: &\qquad \Lambda_1 = \Lambda_3\frac{\gamma}{c}\lambda' - \mu c^{-\gamma} \\
c^e: &\qquad 0 = \mu^e + \Lambda_1 c^e - \Lambda_2 l c^e + \Lambda_3\lambda'
\end{aligned}$$

By constraint 2

$$c^e: \qquad \Lambda_2(1-\beta^e)\mathcal{R}(l,\sigma) = \frac{1}{zh^\alpha}\left(\mu^e + \Lambda_1 c^e + \Lambda_3\lambda'\right)$$

Combine FONCs for c, c^e:

$$\Lambda_2(1-\beta^e)\mathcal{R} = \frac{1}{zh^\alpha}\left[\mu^e - \mu\lambda' + \Lambda_3\lambda'\left(\gamma\frac{c^e}{c} + 1\right)\right]$$

C.1 Log utility

Assume Log utility, $\gamma = 1$:

$$h^{1-\alpha+\psi}c^{\gamma} = \alpha z \left[1 - \frac{\mathcal{R}(1-\beta^e)}{l}\left(1 - \frac{\mu^e}{\mu}\frac{1}{\lambda'}\right)\right]$$

$$h^{1-\alpha+\psi}c^{\gamma} = \alpha z(1-\tau)\left(1 + \frac{l\tau - \left(\frac{\lambda'-\lambda_0}{\lambda'}\right)(1-\beta^e)(1+l\tau)}{l(1-\tau)}\right)$$

where $\lambda_0 := \frac{\mu^e}{\mu}$.

The optimal wage subsidy ς^* is

$$\frac{l\tau - \left(\frac{\lambda'-\lambda_0}{\lambda'}\right)(1-\beta^e)(1+l\tau)}{l(1-\tau)}$$

C.2 Dynamics

Start from the optimal wage subsidy under log utility,

$$\varsigma^* = \frac{\tau}{1-\tau}\left(1 - \hat{\lambda}'(1-\beta^e)\frac{\rho}{\rho-1}\right)$$

where $\hat{\lambda}' := 1 - \frac{\lambda_0}{\lambda'}$. We can solve for $\hat{\lambda}'$,

$$\begin{aligned}
\frac{1}{\lambda'} &= \frac{c}{c^e} \\
&= \frac{y}{c^e} - 1 \\
&= \frac{l}{(1-\beta^e)\rho} - 1 \\
&= \frac{l - (1-\beta^e)\rho}{(1-\beta^e)\rho}
\end{aligned}$$

$$
\begin{aligned}
\hat{\lambda}' &= 1 - \frac{(1-\beta^e)\rho_0}{l_0 - (1-\beta^e)\rho_0} \frac{l - (1-\beta^e)\rho}{(1-\beta^e)\rho} \\
&= 1 - \frac{(1-\beta^e)\rho_0(l - (1-\beta^e)\rho)}{(l_0 - (1-\beta^e)\rho_0)(1-\beta^e)\rho} \\
&= \frac{(l_0 - (1-\beta^e)\rho_0)(1-\beta^e)\rho - (1-\beta^e)\rho_0(l - (1-\beta^e)\rho)}{(l_0 - (1-\beta^e)\rho_0)(1-\beta^e)\rho} \\
&= \frac{l_0 - l\frac{\rho_0}{\rho}}{(l_0 - (1-\beta^e)\rho_0)} \\
&= \left(1 - \frac{l\rho_0}{l_0\rho}\right) \frac{l_0}{(l_0 - (1-\beta^e)\rho_0)}
\end{aligned}
$$

Substituting into the expression for optimal wage subsidies yields

$$
\varsigma^* = \frac{\tau}{1-\tau}\left(1 - \left(\frac{l_0\rho - l\rho_0}{\rho - 1}\right)\frac{(1-\beta^e)}{l_0 - (1-\beta^e)\rho_0}\right) \tag{C2}
$$

C.3 Optimal policy under discretion (Proof of Proposition 2)

Under discretion, the following equilibrium and market clearing conditions are constraints faced by the policymaker. First, the aggregate resource constraint:

$$
zh^\alpha \geq c + c^e \tag{Λ_1}
$$

Second, the policymaker takes entrepreneurial net wealth q^e brought into the period as given. Risk free real interest rates r are fixed by assumption, ultimately implying that policymakers take the opportunity cost of net wealth, n^e, as given. Leverage is therefore increasing in aggregate production:

$$
ln^e = zh^\alpha \tag{Λ_2}
$$

where $\mathcal{R}(l,\sigma) := \rho \ (= 1 + l\tau)$, where τ is a function of l, σ.
The policymaker's objective function can be expressed as follows:

$$
V = \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\mu\beta^t \left(\frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\psi}}{1+\psi}\right) + \mu^e(\beta^e)^t \left(\log c_t^e + \frac{\log g(l_t,\sigma_t)}{1-\beta^e}\right)\right] \tag{C3}
$$

The first order conditions are

$$
\begin{aligned}
h: \quad & \frac{\mu h^{\psi}}{\alpha z h^{\alpha-1}} = \Lambda_1 - \Lambda_2 \\
c: \quad & \frac{\mu}{c^{\gamma}} = \Lambda_1 \\
c^e: \quad & \frac{\mu^e}{c^e} = \Lambda_1 \\
l: \quad & 0 = \Lambda_2 n^e + \frac{\mu^e}{1-\beta^e}\frac{g_l(l_t,\sigma_t)}{g(l_t,\sigma_t)}
\end{aligned}
$$

$$
\frac{c^{\gamma} h^{\psi}}{\alpha z h^{\alpha-1}} = 1 + \frac{c^{\gamma}\mu^e}{\mu}\left(\frac{1}{1-\beta^e}\frac{g_l(l_t,\sigma_t)}{n^e g(l_t,\sigma_t)}\right)
$$

$$
\frac{c^{\gamma} h^{\psi}}{\alpha z h^{\alpha-1}} = 1 + \frac{g_l(l_t,\sigma_t)}{g(l_t,\sigma_t)}\mathcal{R}(l,\sigma)
$$

$$
\frac{c^{\gamma} h^{\psi}}{\alpha z h^{\alpha-1}} = 1 - \mathcal{T}(l,\sigma)
$$

Which is the same as in the competitive allocation in the absence of policy. Given that the policymaker does not introduce a wage subsidy within the period, there is no effect of policy on intertemporal trade, leaving allocations unchanged relative to the competitive equilibrium.

Prudential fiscal stimulus

Online appendix

Alfred Duncan[*] Charles Nolan[†]
University of Kent University of Glasgow

This version: January 2024

This document contains supplementary materials and appendices for the paper Prudential Fiscal Stimulus.

[*]Corresponding author. Email: a.j.m.duncan@kent.ac.uk
[†]Email: charles.nolan@glasgow.ac.uk

Contents

D The equilibrium relationship between leverage and the financial wedge

In deriving the equilibrium relationship between leverage and the financial wedge, it is helpful to spell out in more detail some notation compared to in the main text. There exists a unit measure of entrepreneurs, indexed by i, who enjoy consumption with logarithmic utility,

$$U^e_{ti} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{ej} u^e(c^e_{t+ji}), \tag{D1}$$

where $u^e(c) = \log c$, and $\beta^e < \beta$, entrepreneurs are less patient than households.[1] Entrepreneurs undertake projects with binary risky outcomes. The individual output of entrepreneur i is

$$y_{ti} = \theta_{ti} z_t K^{1-\psi}_{ti} h^{\psi}_{ti}, \tag{D2}$$

where θ_{it} is an idiosyncratic shock drawn from $\theta_{ti} \in \{\underline{\theta}_t, \overline{\theta}_t\}$ where $\underline{\theta}_t < \overline{\theta}_t$. These two states occur with probabilities $\underline{\pi}$ and $\overline{\pi}$ and their expectation is equal to one, $\underline{\pi}\underline{\theta}_t + \overline{\pi}\overline{\theta}_t = 1$. Define $\sigma^\theta_t = \overline{\theta}_t - \underline{\theta}_t$ which has a law of motion $\log \sigma^\theta_t = \rho_\sigma \log \sigma^\theta_{t-1} + \epsilon_{\sigma t}$, where $\epsilon_{\sigma t}$ is a white noise process with standard deviation σ_ξ. Denote the expectation of output for entrepreneur i in period t conditional upon z_t by $\bar{Y}_{ti} = z_t K^{1-\psi}_{ti} h^{\psi}_{ti}$. The variable z_t is an aggregate total factor productivity shock following the law of motion $\log z_t = \rho_z \log z_{t-1} + \epsilon_{zt}$, where ϵ_{zt} is a white noise process with standard deviation σ_z. Aggregate shocks are observable at the beginning of the period, whereas the unobservable shock is revealed to the entrepreneur at the end of the period. Capital employed by the entrepreneur is denoted in period t is K_{ti}, and h_{ti} is labour hired by the entrepreneur from the household sector.

At the beginning of each period, entrepreneurs borrow capital q_{ti} from households. Loan contracts specify the interest rate paid in good states, as well as the recovery rate returned to financial intermediaries in bad states. Capital inputs into entrepreneur i's project include the entrepreneur's initial capital holdings and further capital borrowed.

$$K_{ti} = q^e_{ti} + q_{ti}, \tag{D3}$$

where q^e_t is the capital held by the entrepreneur at the beginning of the period. Entrepreneurs fund consumption and future capital holdings out of the sum of project

[1] Entrepreneurs enjoy a greater return on savings than households in the model. Their reduced discount factor is required to ensure that the ratio of entrepreneurial and household wealth is constant in the steady state.

revenues and current capital holdings, after repaying loans and paying workers' wages,

$$Q_t q^e_{ti} + c^e_{ti} = y_{ti}(\theta_{ti}) + Q_t(1-\delta)q^e_{ti} - q_{ti}\hat{r}_{ti}(\theta_{ti}, \vartheta_{ti}) - w_t h_{ti}. \tag{D4}$$

where δ is the depreciation rate of capital. Lagrange multipliers $\lambda^e_{ti}(\theta_{ti}, \vartheta_{ti})$ are attached to each of the state contingent accumulation constraints. Note that capital rental payments $\hat{r}_{ti}(\theta_{ti}, \vartheta_{ti})$ are contingent on the idiosyncratic shock θ_{ti} as well as any audit signal obtained by the financial intermediary, $\vartheta_{ti} \in \{\underline{\vartheta}, \overline{\vartheta}\}$. Audit signals are distributed as follows: $P(\overline{\vartheta}|\overline{\theta}) = 1, P(\overline{\vartheta}|\underline{\theta}) = \eta, P(\underline{\vartheta}|\underline{\theta}) = 1 - \eta$. In other words, high types are always correctly identified by the audit, but with probability η low types are incorrectly tagged as high types. Consequently, the unconditional probabilities of the three possible outcomes are as follows:

$$P(\underline{\theta}, \underline{\vartheta}) = \underline{\pi}(1-\eta), \qquad P(\underline{\theta}, \overline{\vartheta}) = \underline{\pi}\eta, \quad \text{and} \quad P(\overline{\theta}, \emptyset) = \overline{\pi}. \tag{D5}$$

Duncan and Nolan (2019) show that when audit costs are not too high and auditing is imperfect, defaultable debt contracts with deterministic audit strategies are constrained efficient. Contracts are only contingent on reports and audit signals within the current period. This restriction is referred to as the *anonymity* constraint. Once repayments on current period loans are made, entrepreneurs are considered to become anonymous, and their future actions in other markets cannot be used as evidence of past false reports.

Thus to access outside investment, contracts are subject to two constraints. First, repayments following overturned low reports must exceed those following high reports,

$$\hat{r}(\underline{\theta}, \overline{\vartheta}) \geq \hat{r}(\overline{\theta}, \emptyset). \tag{D6}$$

This is the incentive compatibility constraint, and we attach to it the Lagrange multiplier μ. Second, expected loan repayments must exceed the sum of the households' opportunity cost of funds, $r_t q_{ti}$, and expected audit costs, $\underline{\pi}\kappa K_{ti}$:

$$\sum_{(\theta_{ti}, \sigma^\theta_{ti})} P(\theta_{ti}, \vartheta_{ti})\hat{r}_t(\theta_{ti}, \vartheta_{ti})q_{ti} \geq r_t q_{ti} + \underline{\pi}\kappa K_{ti}. \tag{D7}$$

Equation D7 describes the households' participation constraint, to which is attached the Lagrange multiplier ν. Both the incentive compatibility and participation constraints will be binding under efficient contracts.

Given the assumptions of homothetic (log) utility and constant returns to scale production, along with the assumption that entrepreneurs' financial market contracts are anonymous across markets, we can study the entrepreneurs' within period external finance problem separately from their across period aggregate risk sharing problem. The within period problem is to maximise the following objective,

$$\max \mathbb{E}_t \sum_{j=0}^{\infty} \beta^{ej} u^e(c^e_{t+ji})$$

subject to

$$\begin{aligned}
Q_t q^e_{ti} + c^e_{ti} &= y_{ti}(\theta_{ti}) + Q_t(1-\delta)q^e_{ti} - q_{ti}\hat{r}_{ti}(\theta_{ti}, \vartheta_{ti}) - w_t h_{ti} \\
y_{ti} &= \theta_{ti} z_t K^{1-\psi}_{ti} h^{\psi}_{ti} \\
K_{ti} &= q^e_{ti} + q_{ti} \\
\sum_{(\theta_{ti}, \sigma^{\theta}_{ti})} P(\theta_{ti}, \vartheta_{ti})\hat{r}_t(\theta_{ti}, \vartheta_{ti}) q_{ti} &\geq r_t q_{ti} + \underline{\pi}\kappa K_{ti} \\
\hat{r}(\underline{\theta}, \overline{\vartheta}) &\geq \hat{r}(\overline{\theta}, \emptyset)
\end{aligned}$$

and initial conditions on state variables. The Lagrange multiplier on the budget constraint is denoted $\lambda^e_{ti}(\theta_{ti}, \vartheta_{ti})$.

First order necessary conditions

Given this set up, the first order necessary conditions follow:

$$\begin{aligned}
h_{ti}:\ & 0 = \mathbb{E}_t \lambda^e_{ti}(\theta_{ti}, \vartheta_{ti}) \left[Y_{Nti}(\theta_{ti}) - w_t\right], && \text{(D8)} \\
q_{ti}:\ & 0 = \mathbb{E}_t \lambda^e_{ti}(\theta_{ti}, \vartheta_{ti})[Y_{Kti}(\theta_{ti}) - \hat{r}_{ti}(\theta_{ti}, \vartheta_{ti})] \\
& \quad + \nu_{ti}\left[\mathbb{E}_t \hat{r}_t(\theta_{ti}, \vartheta_{ti}) - r_t - \underline{\pi}\kappa\right] && \text{(D9)} \\
c^e_{ti}(\theta_{ti}, \vartheta_{ti}):\ & 0 = u^{e\prime}(c^e_{ti}(\theta_{ti}, \vartheta_{ti})) - \lambda^e_{ti}(\theta_{ti}, \vartheta_{ti}), && \text{(D10)} \\
K^e_{t+1i}(\theta_{ti}, \vartheta_{ti}):\ & 0 = -Q_t \lambda^e_{ti}(\theta_{ti}, \vartheta_{ti}) \\
& \quad + \beta^e \mathbb{E}_{t+}\left[\lambda^e_{t+1i}(\theta_{t+1i}, \vartheta_{t+1i})\left(Y_{Kt+1i} + Q_{t+1}(1-\delta)\right) - \nu_{t+1i}\pi_l\kappa\right] \\
& && \text{(D11)} \\
\hat{r}_t(\underline{\theta}, \underline{\vartheta}):\ & 0 = -P(\underline{\theta}, \underline{\vartheta})\lambda^e_{ti}(\underline{\theta}, \underline{\vartheta})q_{ti} + \nu_{ti}P(\underline{\theta}, \underline{\vartheta})q_{ti} && \text{(D12)} \\
\hat{r}_t(\underline{\theta}, \overline{\vartheta}):\ & 0 = -P(\underline{\theta}, \overline{\vartheta})\lambda^e_{ti}(\underline{\theta}, \overline{\vartheta})q_{ti} + \nu_{ti}P(\underline{\theta}, \overline{\vartheta})q_{ti} + \mu_{ti} && \text{(D13)} \\
\hat{r}_t(\overline{\theta}, \emptyset):\ & 0 = -P(\overline{\theta}, \emptyset)\lambda^e_{ti}(\overline{\theta}, \emptyset)q_{ti} + \nu_{ti}P(\overline{\theta}, \emptyset)q_{ti} - \mu_{ti} && \text{(D14)}
\end{aligned}$$

Let $Y_{jti}(\theta_{ti})$ denotes the derivative of output with respect to factor j for entrepreneur i in period t given idiosyncratic shock realisation θ_{ti}. Also, let $\bar{Y}_{jti}$ denote the expectation of the derivative of output with respect to factor j for entrepreneur i in period t over idiosyncratic shock realisations θ_{ti}. A full analysis of these conditions is developed in Duncan and Nolan (2019, 2022) but their essential intuition will be set out. We then turn specifically to derive the labour wedge of inefficiency and its equilibrium relationship to leverage.

D.1 Risk across states

The final three first order conditions above describe how the entrepreneurs' marginal utility (captured by $\lambda^e_{ti}(\theta, \vartheta)$) varies across states. Entrepreneurs can vary loan repayment rates across states $\hat{r}(\theta, \vartheta)$ in order to attempt to reduce variations in λ^e_{ti} across states. Entrepreneurs' ability to reduce variations in λ^e_{ti} across states is limited by the entrepreneurs' incentive compatibility constraint (D6). The incentive compatibility constraint is binding under an efficient contract ($\mu_{ti} > 0$) resulting in varying marginal utilities across idiosyncratic states $\lambda^e_{ti}(\underline{\theta}, \overline{\vartheta}) > \lambda^e_{ti}(\underline{\theta}, \underline{\vartheta}) > \lambda^e_{ti}(\overline{\theta}, \emptyset)$, showing that it is necessary but costly for a low type to be identified erroneously as a high type. In addition to the option of reduced repayments following successful audits, entrepreneurs can mitigate project risk by reducing the size of projects, relative to the size which maximises expected profits. This precautionary reduction in the size of projects translates into reductions in the quantities of capital and labour demanded compared with first best efficient allocations. The full aggregate risk-sharing condition used in the paper is derived in Appendix A.

D.2 Labour and capital market wedges

Equation D8 describes the entrepreneur's first order necessary condition for labour demanded. Note the presence of an expectations operator: wage contracts are determined at the beginning of the period, *prior* to the revelation of idiosyncratic shocks and so wages are not necessarily equal to the marginal product of labour ex post. The entrepreneurs weight deviations between wages and labour's marginal product according to the likelihood of states and the entrepreneurs' shadow value of wealth conditional upon potential states. The entrepreneurs weight deviations between the ex post marginal product of labour and wages more highly in bad states, when their marginal utility is high. Moreover, averaging over entrepreneurs, the expected marginal product of labour does not equal the wage rate, even in the absence of aggregate risk. The labour market wedge is sensitive to both the idiosyncratic distribution of project outcomes, as well as financial variables including the accuracy of audit signals (η).

Similar considerations impact the capital market; again see Duncan and Nolan (2022) for more details. Here we will focus on labour but as regards capital/inside wealth, we note that there is a difference, a wedge, between the expected marginal product of capital and the capital rental rate in just the same way as there is for the labour market optimality.

Let the labour and capital market wedges be defined as follows,

$$\tau_{Nti} = \frac{\bar{Y}_{Nti} - w_t}{\bar{Y}_{Nti}} \quad \text{and} \tag{D15}$$

$$\tau_{Kti} = \frac{\bar{Y}_{Kti} - r_t}{\bar{Y}_{Kti}}. \tag{D16}$$

It then follows that

$$\frac{\lambda^e_{ti}(\underline{\theta}, \overline{\vartheta})}{\lambda^e_{ti}(\underline{\theta}, \underline{\vartheta})} = 1 + \frac{\tau_{Nti}}{P(\underline{\theta}, \overline{\vartheta})\sigma^\theta_t}, \tag{D17}$$

$$\frac{\lambda^e_{ti}(\overline{\theta}, \emptyset)}{\lambda^e_{ti}(\underline{\theta}, \underline{\vartheta})} = 1 - \frac{\tau_{Nti}}{P(\overline{\theta}, \emptyset)\sigma^\theta_t} \quad \text{and} \tag{D18}$$

$$\tau_{Nti} = \tau_{Kti} - \frac{\overline{\pi}\kappa}{\bar{Y}_{Kti}}. \tag{D19}$$

To go from Equation D8 to the first of these expressions note that Equation D8 may be written as

$$0 = P(\underline{\theta}, \underline{\vartheta})\lambda^e_{ti}(\underline{\theta}, \underline{\vartheta})Y_{Nti}(\underline{\theta}) + P(\underline{\theta}, \underline{\vartheta})\lambda^e_{ti}(\underline{\theta}, \overline{\vartheta})Y_{Nti}(\underline{\theta}) + P(\overline{\theta}, \emptyset)\lambda^e_{ti}(\overline{\theta}, \emptyset)Y_{Nti}(\overline{\theta}) - \mathbb{E}_t\lambda^e_{ti}(\theta_{ti}, \vartheta_{ti})w_t.$$

A similar re-writing for the first-order condition for q_{ti} is also possible. Reorganising these expressions delivers the three expressions immediately above. These equations relate the entrepreneurs' marginal rates of substitution for consumption across project outcomes to the labour market wedge. The final equation relates the labour market wedge to the capital market wedge. The labour market wedge (the left hand side, τ_{Nti}) is less than the capital market wedge (τ_{Kti}). For entrepreneurs, it is efficient to reduce both labour and capital demanded in order to mitigate project risk. The difference between the two wedges results follows as a result of auditing costs, which are increasing in the capital factor but not in the labour factor.

Factor income

The previous subsection showed how the entrepreneurs' inability to share idiosyncratic productive risk with households results in positive labour and capital market wedges; household's productive factor inputs are lower than their expected marginal products. Aggregating over entrepreneurs, we can combine the definitions of the factor market wedges (D15) and (D16) with the optimality condition relating the two wedges (D19) to derive the composition of output (national income) in terms of factor income:

$$Y_t = w_t h_t + r_t[q_t + q_t^e] + y_t \tau_{Nt} + \pi_l \kappa K_t. \tag{D20}$$

The first two terms on the right hand side typically form the decomposition of factor income in a frictionless model. When entrepreneurs are compensated for risk, there are two additional terms: the compensation for risk bearing earned by entrepreneurs is captured by $y_t \tau_{Nt}$. The fourth term, $\pi_l \kappa K_t$, captures the resource costs of audits.

D.3 Labour market wedge and equilibrium leverage

It is now possible to solve for the labour market wedge in terms of leverage and project uncertainty. First, substitute the national income equation to eliminate $w_t h_t$,

$$\frac{\lambda_{ti}^e(\underline{\theta}, \overline{\vartheta})}{\lambda_{ti}^e(\underline{\theta}, \underline{\vartheta})} = \frac{(\underline{\theta} - 1 + \tau_{Nt})\bar{Y}_t + [Q_t(1-\delta) + r_t]q_{ti}^e - q_{ti}\hat{r}_{ti}(\underline{\theta}, \underline{\vartheta}) + r_t q_t + \pi_l \kappa K_t}{(\underline{\theta} - 1 + \tau_{Nt})\bar{Y}_t + [Q_t(1-\delta) + r_t]q_{ti}^e - q_{ti}\hat{r}_{ti}(\overline{\theta}, \emptyset) + r_t q_t + \pi_l \kappa K_t} \tag{D21}$$

$$\frac{\lambda_{ti}^e(\overline{\theta}, \emptyset)}{\lambda_{ti}^e(\underline{\theta}, \underline{\vartheta})} = \frac{(\underline{\theta} - 1 + \tau_{Nt})\bar{Y}_t + [Q_t(1-\delta) + r_t]q_{ti}^e - q_{ti}\hat{r}_{ti}(\underline{\theta}, \underline{\vartheta}) + r_t q_t + \pi_l \kappa K_t}{(\overline{\theta} - 1 + \tau_{Nt})\bar{Y}_t + [Q_t(1-\delta) + r_t]q_{ti}^e - q_{ti}\hat{r}_{ti}(\overline{\theta}, \emptyset) + r_t q_t + \pi_l \kappa K_t} \tag{D22}$$

Note that $\underline{\pi}\underline{\theta} + \overline{\pi}\overline{\theta} = 1$, which implies $\underline{\theta} = 1 - \overline{\pi}\sigma_t^\theta$, and $\overline{\theta} = 1 + \underline{\pi}\sigma_t^\theta$. We now proceed to re-write the right hand side of these expressions in terms of uncertainty and project risk. Then, we use the definition of the wedges to substitute out for the marginal rate of substitutions (ratios of Lagrange multipliers). The result is an expression relating equilibrium leverage to the labour wedge.

First, then, rather than working with specific interest rates $\hat{r}(\cdot)$ and project returns θ, we write equations in terms of project risk $\sigma_t^\theta = \overline{\theta} - \underline{\theta}$, and risk sharing $[\hat{r}_{ti}(\overline{\theta}, \emptyset) - \hat{r}_{ti}(\underline{\theta}, \underline{\vartheta})]$. From the households' participation constraint,

$$q_{ti}[(\overline{\pi} + \underline{\pi}\eta)\hat{r}_{ti}(\overline{\theta}, \emptyset) + \underline{\pi}(1-\eta)\hat{r}_{ti}(\underline{\theta}, \underline{\vartheta})] = r_t q_t + \pi_l \kappa K_t$$

one is able to re-write $\hat{r}_{ti}(\overline{\theta},\emptyset),\hat{r}_{ti}(\underline{\theta},\underline{\vartheta})$ in terms of required returns and risk:

$$\hat{r}_{ti}(\overline{\theta},\emptyset)=r_t+\pi_l\kappa\frac{K_t}{q_{ti}}+\underline{\pi}(1-\eta)(\hat{r}_{ti}(\overline{\theta},\emptyset)-\hat{r}_{ti}(\underline{\theta},\underline{\vartheta}))$$

$$\hat{r}_{ti}(\underline{\theta},\underline{\vartheta})=r_t+\pi_l\kappa\frac{K_t}{q_{ti}}-(\overline{\pi}+\underline{\pi}\eta)(\hat{r}_{ti}(\overline{\theta},\emptyset)-\hat{r}_{ti}(\underline{\theta},\underline{\vartheta})).$$

Substituting these back into (D21,D22) and rearranging shows that

$$R_tQ_{t-1}\frac{q_{ti}}{q_{ti}}=(\overline{\pi}\sigma_t^{\theta}-\tau_{Nt})\frac{\bar{Y}_t}{q_{ti}}-\left[\frac{1}{1-\lambda^e_{ti}(\underline{\theta},\overline{\vartheta})/\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}-\underline{\pi}(1-\eta)\right](\hat{r}_{ti}(\overline{\theta},\emptyset)-\hat{r}_{ti}(\underline{\theta},\underline{\vartheta}))$$

$$\begin{aligned}R_tQ_{t-1}\frac{q^e_{ti}}{q_{ti}}&=(\overline{\pi}\sigma_t^{\theta}-\tau_{Nt})\frac{\bar{Y}_t}{q_{ti}}+\frac{\lambda^e_{ti}(\overline{\theta},\emptyset)/\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}{1-\lambda^e_{ti}(\overline{\theta},\emptyset)/\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}\frac{\bar{Y}_t}{q_{ti}}\sigma_t^{\theta}\\&-\left[\frac{1}{1-\lambda^e_{ti}(\overline{\theta},\emptyset)/\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}-\underline{\pi}(1-\eta)\right](\hat{r}_{ti}(\overline{\theta},\emptyset)-\hat{r}_{ti}(\underline{\theta},\underline{\vartheta}))\end{aligned}$$

where $R_t\equiv\frac{r_t+(1-\delta)Q_t}{Q_{t-1}}$. And we equate the right hand sides to solve:

$$\frac{\bar{Y}_t\sigma_t^{\theta}}{q_{ti}(\hat{r}_{ti}(\overline{\theta},\emptyset)-\hat{r}_{ti}(\underline{\theta},\underline{\vartheta}))}=\left[\frac{\frac{\lambda^e_{ti}(\underline{\theta},\overline{\vartheta})}{\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}/\frac{\lambda^e_{ti}(\overline{\theta},\emptyset)}{\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}-1}{\frac{\lambda^e_{ti}(\underline{\theta},\overline{\vartheta})}{\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}-1}\right]$$

Note that $\frac{\lambda^e_{ti}(\underline{\theta},\overline{\vartheta})}{\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}>1$, and $\frac{\lambda^e_{ti}(\overline{\theta},\emptyset)}{\lambda^e_{ti}(\underline{\theta},\underline{\vartheta})}<1$, making the right hand side strictly greater than 1. The left hand side is the ratio of productive risk to the possible amount of risk sharing following loan restructuring. The equation shows how an increase in the ratio of productive risk to risk sharing increases the entrepreneurs' marginal rates of substitution for consumption across idiosyncratic states. These marginal rates of substitution in turn determine entrepreneurs' precautionary reductions in wage and capital hiring compared with the first best efficient levels, through the wedges specified in equations (D17) and (D18). Substituting these factor wedges in place of the marginal rates of substitution yields

$$q_{ti}[\hat{r}_{ti}(\overline{\theta},\emptyset)-\hat{r}_{ti}(\underline{\theta},\underline{\vartheta})]=\bar{Y}_{ti}\left[\frac{\overline{\pi}\sigma_t^{\theta}-\tau_{Nti}}{\overline{\pi}+\underline{\pi}\eta}\right]\tag{D23}$$

Now we can use this solution to solve for the leverage ratio

$$\frac{\bar{Y}_{ti}}{R_tQ_{t-1}q^e_{ti}}\equiv l_t=\frac{(\overline{\pi}+\underline{\pi}\eta)\tau_{Nti}}{[\overline{\pi}\sigma_t^{\theta}-\tau_{Nti}][\underline{\pi}\eta\sigma_t^{\theta}+\tau_{Nti}]},\tag{= (RHS)}$$

Finally,re-writing this expression yields the expression in the main text

$$
\begin{aligned}
\tau_{Nti} = & \frac{\left(\sqrt{\frac{\overline{\pi}}{\underline{\pi}}} - \sqrt{\frac{\underline{\pi}}{\overline{\pi}}}\eta\right)\sigma l_t - (\overline{\pi} + \underline{\pi}\eta)}{2l_t} + \\
& \frac{\sqrt{\left[\left(\sqrt{\frac{\overline{\pi}}{\underline{\pi}}} - \sqrt{\frac{\underline{\pi}}{\overline{\pi}}}\eta\right)\sigma l_t - (\overline{\pi} + \underline{\pi}\eta)\right]^2 + 4\eta l_t \sigma^2}}{2l_t} \\
& \equiv \mathcal{T}(l_t, \sigma),
\end{aligned}
$$

where we have used the fact that $\sigma_t^\theta \equiv \left(\overline{\theta} - \underline{\theta}\right) = \sigma / \sqrt{\overline{\pi}\underline{\pi}}$ where σ is the standard deviation of the idiosyncratic shock, θ.

E Model equations in full

Helpful notation

$$n^e := (1+r)q^e$$

In words, entrepreneurial net wealth within the period redenominated in terms of the consumption units that could be purchased with no risk at the end of the period with that wealth.

$$\rho := \frac{\mathbb{E}_\Theta[R(\theta,s)]}{1+r}$$

Production and aggregate demand

$$y = zh^\alpha$$

$$y = c + c^e \tag{E1}$$

Intratemporal financial contracts

$$l = \frac{y}{n^e} \tag{E2}$$

$$\tau = \frac{\left(\sqrt{(\bar{\pi}/\underline{\pi})} - \sqrt{(\underline{\pi}/\bar{\pi})}\eta\right) l\sigma + (\bar{\pi} + \underline{\pi}\eta)}{2l} + \frac{\sqrt{\left[\left(\sqrt{(\bar{\pi}/\underline{\pi})} - \sqrt{(\underline{\pi}/\bar{\pi})}\eta\right) l\sigma + (\bar{\pi} + \underline{\pi}\eta)\right]^2 + 4\eta l^2\sigma^2}}{2l} \tag{E3}$$

Entrepreneurs' optimal consumption

$$c^e = (1-\beta^e)\rho n^e$$

Factor prices

$$\rho = 1 + l\tau \tag{E4}$$

$$w = \frac{\alpha y}{h}(1-\tau)$$

Labor supply

$$\frac{h^{\psi}}{c^{-\gamma}} = w$$

Risk sharing

$$\lambda' = \frac{c^{-\gamma}}{(c^{e})^{-1}} \tag{E5}$$

$$\lambda' = \lambda \frac{\beta^{e}}{\beta} \rho \tag{E6}$$

F Welfare calculations

Let

$$\bar{\gamma} := \frac{\sqrt{(\bar{\pi}/\underline{\pi})}\sigma}{\sqrt{(\bar{\pi}/\underline{\pi})}\sigma - \tau}, \qquad \underline{\gamma} := \frac{\sqrt{(\underline{\pi}/\bar{\pi})}\eta\sigma}{\sqrt{(\underline{\pi}/\bar{\pi})}\eta\sigma + \tau}$$

Under optimal financial contracts, the following conditions hold for entrepreneurs:

$$c^e(\underline{\theta}, \bar{\phi}) = \underline{\gamma} c^e(\underline{\theta}, \underline{\phi}) \tag{F1}$$

$$c^e(\bar{\theta}) = \bar{\gamma} c^e(\underline{\theta}, \underline{\phi}). \tag{F2}$$

$$\bar{c^e} = \bar{\pi} c^e(\bar{\theta}) + \underline{\pi}(1-\eta) c^e(\underline{\theta}, \underline{\phi}) + \underline{\pi}\eta c^e(\underline{\theta}, \bar{\phi})$$

Solving the above system, we can derive the realised growth rates of consumption of an individual entrepreneur as a function of the entrepreneurs' individual shock realisations, the mean growth rate of consumption, the factor price wedge τ and idiosyncratic risk σ. Without loss of generality, we refer to these realised consumption growth rates as

$$g'(\bar{\theta}) := \frac{c^{e\prime}(\bar{\theta})}{\bar{c}} = \mathcal{G}(\bar{\theta}, \bar{c^e}', \bar{c^e}, \tau, \sigma)$$

The arguments $\bar{c^e}', \bar{c^e}, \tau, \sigma$ are aggregates, permitting us to evaluate the lifetime value of a representative entrepreneur who enters at time zero with wealth equal to mean entrepreneurial wealth.

It follows that the consumption of an entrepreneur in period T with time zero consumption $\bar{c^e}_0$ can be expressed as follows

$$c^e_t = \bar{c^e}_0 \prod_{t=1}^{T} \mathcal{G}(\theta_t, \bar{c^e}_t, \bar{c^e}_{t-1}, \tau_t, \sigma_t) \qquad \theta_t \in \{\bar{\theta}, (\underline{\theta}, \underline{\phi}), (\underline{\theta}, \bar{\phi})\}.$$

Recall that we assume entrepreneurs enjoy consumption with log utility, and discount future period utility with time preference parameter β^e. We can therefore derive the following expression for the lifetime value of this entrepreneur:

$$(1-\beta^e)v^e = \log \bar{c^e}_0 + \mathbb{E}\sum_{t=1}^{\infty}(\beta^e)^t \log \mathcal{G}(\theta_t, \bar{c^e}_t, \bar{c^e}_{t-1}, \tau_t, \sigma_t) \tag{F3}$$

The expectation in the expression above is an expectation over future realisations of both aggregate and individual specific states. The above expression allows us to solve for the expectation over individual specific states with certainty, for any realised path of aggregate states. We approximate the expectation over aggregate states from simulated paths of the aggregate economy.

F.1 Solving for consumption growth rates

$$\mathcal{G}((\underline{\theta}, \underline{\phi}), \bar{c^e}_t, \bar{c^e}_{t-1}, \tau_t, \sigma_t) = \frac{\bar{c^e}_t}{\bar{c^e}_{t-1}} \frac{1}{\bar{\pi}\bar{\gamma} + \underline{\pi}(1-\eta) + \underline{\pi}\eta\underline{\gamma}} \tag{F4}$$

We can combine Equation F4 with (F1,F2) to find the consumption growth rates conditional upon different realisations of (θ, ϕ).

G Estimation

Both the nonlinear and loglinearised models are estimated using the No-U-Turn Sampler of Hoffman and Gelman (2011), as implemented in Turing (Ge et al., 2018).

G.1 Nonlinear flexible price model

For the nonlinear model, in order to retain differentiability of the log-likelihood with respect to deep model parameters, we treat the iid component of innovations in TFP and the risk shock as parameters. This increases the number of estimated parameters, but this has little computational cost due to the efficient scaling of Hamiltonian Monte Carlo samplers in the parameter space.

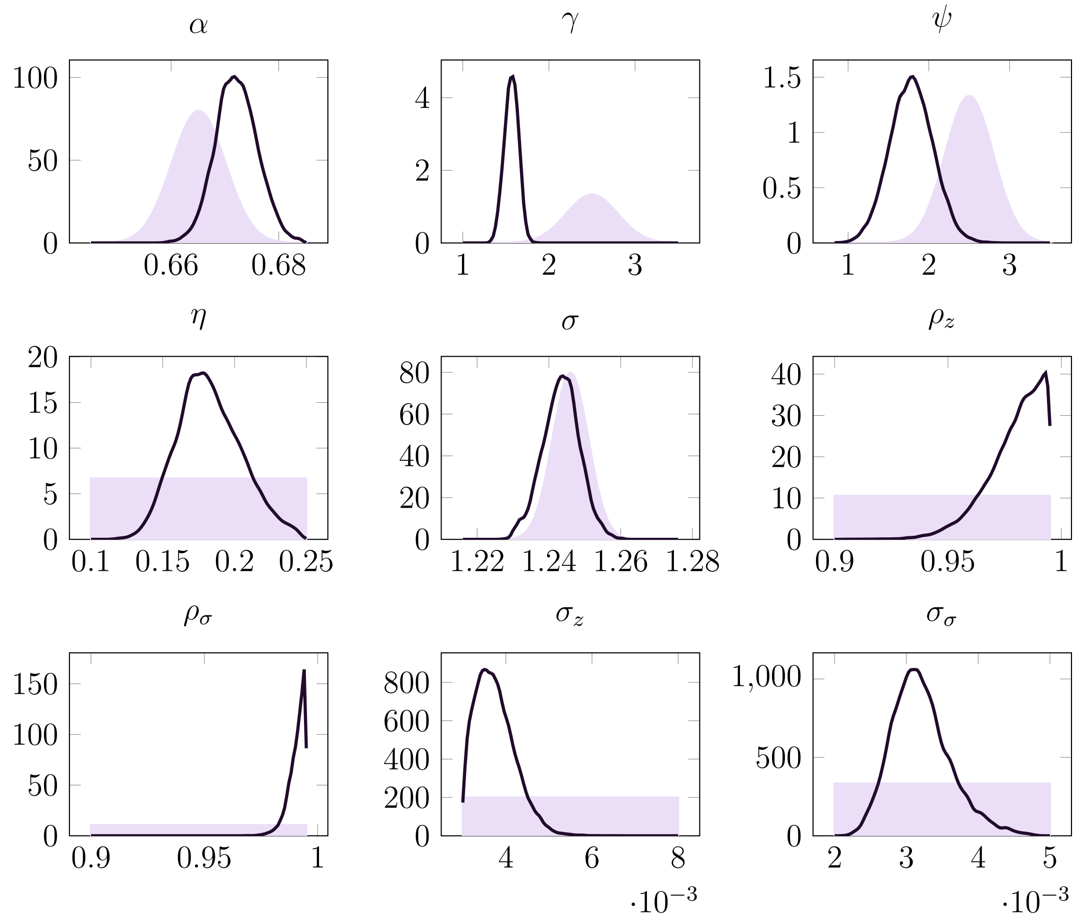
The data we use to estimate the model is the output-employment ratio and the labor share of income for the US from 1984Q1 to 2019Q4. The output-employment ratio is constructed as the ratio of real GDP per capita (Fred: `A939RX0Q048SBEA`) to the employment population ratio (Fred: `EMRATIO`). The labor share is constructed as the ratio of the total Compensation of Employees (Fred: `GDICOMP`) to GDP (Fred: `GDP`). We do not remove trends from the log-level data, but allow for trends in the model's TFP process. These trends are later removed when we use the model for policy analysis.[2]

Figure 1 presents the prior and posterior distributions for model structural parameters. We have omitted the distributions relating to trends and innovations in shock processes, as these are not carried over to our policy analysis. For the production labor share parameters α and the steady state uncertainty parameter σ, our priors are tightly informed by pre-sample steady state values, and, given that we do not detrend our time series data, the posterior estimates are also predominantly informed by the steady state ratios present in the time series data. Given α, the steady state labor ratio is determined by the combination of the risk shock σ and the audit quality parameter η. The labor share is decreasing in both parameters. Combinations with high values of η and lower values of σ tend to have a lower semi-elasticity of the labor wedge with respect to leverage, and so tend to support higher volatility of leverage. The parameter with the largest quantitative impact on our results is ρ_z;

[2] In earlier exerises, we also allowed for trends in the risk process but the estimated trends were very small, but the inclusion of these trends reduced numerical stability.

this is discussed in Section 4.

Figure 1: Prior (■) and posterior (—) parameter distributions
for the benchmark nonlinear model



G.2 Log-linearised sticky price model

The log-linearised model of Section I is also estimated using the Hamiltonian Monte Carlo sampler. Efficient differentiation of the model solution step follows Duncan (2021).

For estimation, the model is closed with the following Taylor rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_y (y_t - \varrho z_t)) + \varepsilon_{it},$$

Table 1: Calibrated parameters

Parameter	Value	Data target (Fred ID)
Real interest rate r_A	1.83% p.a.	90 day treasury bills (DTB3); Consumer Price Index: Total All Items (CPALTT01USQ657N)
Leverage, l	0.12	Nonfinancial Business, Gross Value Added (NNBGAVQ027S; A455RC1Q027SBEA); Nonfinancial Business, Net Worth (TNWBSNNB; TNWMVBSNNCB); DTB3; CPALTT01USQ657N
Equity risk premium, ρ	12.0% p.a.	Nonfinancial Business, Net Operating Surplus (NNBBOSQ027S; W326RC1Q027SBEA); TNWBSNNB; TNWMVBSNNCB; DTB3; CPALTT01USQ657N
Labour share, LS	0.62	Share of Labour Compensation in GDP LABSHPUSA156NRUG
Derived parameter		Transformation
Time preference, β	0.995	$1/(1 + r_A/400)$
Entrepreneur time preference, β^e	0.968	β/ρ
Steady state factor wedge, τ_0	0.248	$(\rho - 1)/l$
Capital production weight, α	0.182	$1 - \text{LS}/(1 - \tau)$
Distribution of consumption, Δ	0.384	$\beta(1 - \beta^e)/(l - \beta(1 - \beta^e))$

where $\varrho := (1 + \psi)/(\gamma(1 - \alpha) + \psi + \alpha)$.

G.2.1 Calibrated parameters

We calibrate the steady state risk free real interest rate, leverage ratio, equity risk premium, and labour share. Calibration targets are presented in Table 1.

The distribution parameter Δ is not targeted, but can be compared with empirical estimates. We use the 2019 wave of the University of Michican Panel Study of Income Dymamics (PSID) to estimate the distribution of consumption between entrepreneurs other households. Classifying business owners as households yields an entrepreneurial consumption ratio of 0.264, while including households with managerial occupations yields an entrepreneurial consumption ratio of 0.470. Our model predicted value of 0.384 lies within this range.

G.2.2 Estimated parameters

The model is estimated using GDP deflator growth (Fred: `USAGDPDEFQISMEI`), output per capita growth (Fred: `A939RX0Q048SBEA`) and nominal interest rates (Fred: `DTB3`) for the US from 1984Q1 to 2019Q4.

Figure 2: Prior (■) and posterior (—) parameter distributions for the sticky price log-linearised model. Selected parameters.

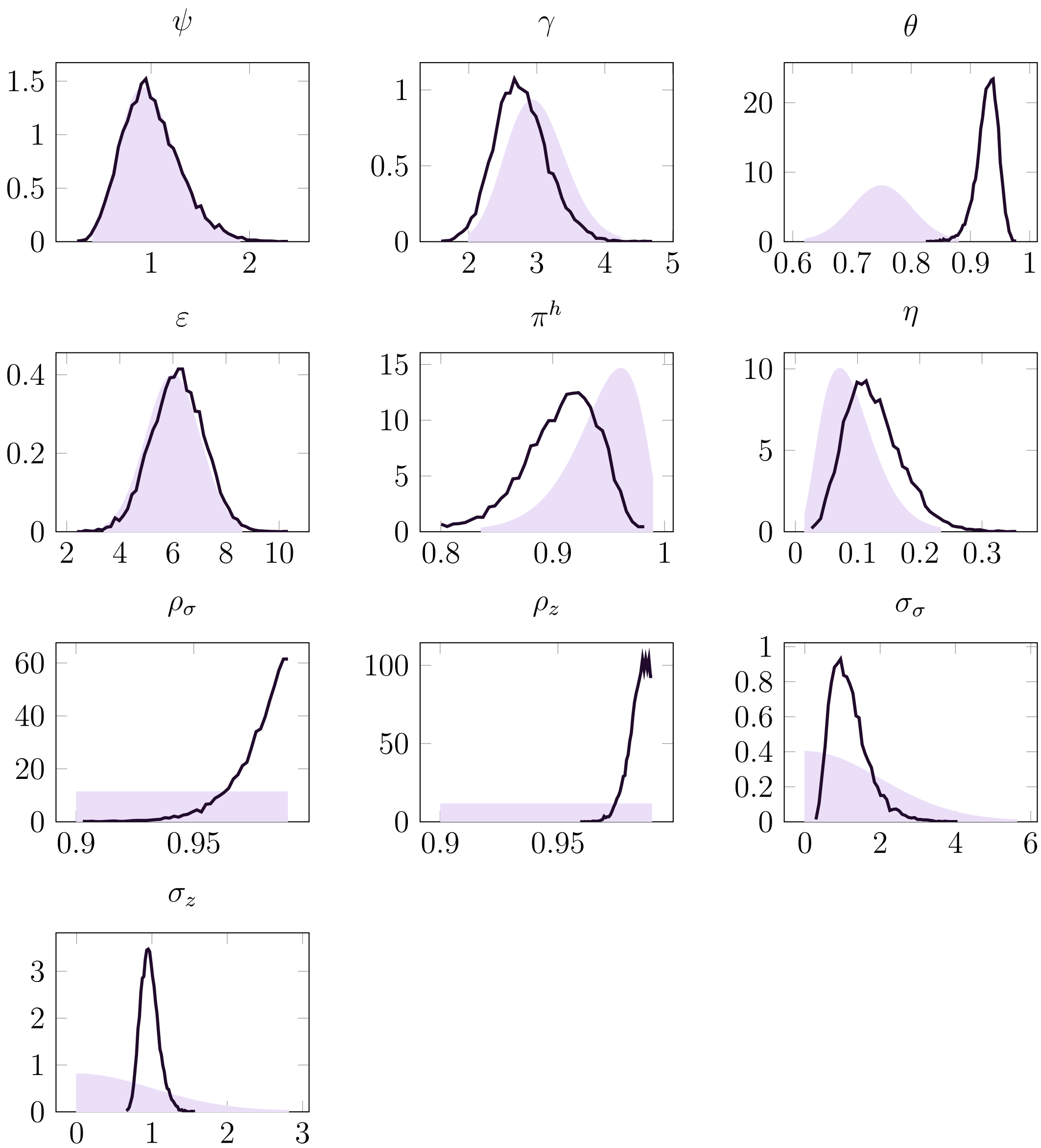


Figure 2 presents the prior and posterior distributions, only showing the estimated parameters that remain in the optimal policy programmes studied in Section 5 (parameters relating to the Taylor rule and measurement error shocks are not displayed).

H Derivations of key model equations

H.1 Derivations of Equations 4,5, and 6

The national income identity is

$$y = wh + kr^k + \hat{\tau}y$$

where wh is the total wage bill, r^k is the capital rental rate, k is the stock of capital, and $\hat{\tau}y$ is defined as factorless income. Factorless income is absorbed by the entrepreneurs, who own the firms.

The total income of entrepreneurs is therefore

$$rn^e(1+\rho) = rn^e + \hat{\tau}y,$$

which, when combined with (4), rearranges to yield

$$\frac{\mathbb{E}_\Theta[R(\theta, s)]}{1+r} = 1 + l\hat{\tau}.$$

Capital rental and labour hiring decisions are made within the period. The entrepeneur's first order conditions for their factor market purchases are

$$1 = \frac{\mathbb{E}_\Theta[\Upsilon(\theta)\theta f_h(k,h)]}{w} = \frac{\mathbb{E}_\Theta[\Upsilon(\theta)\theta f_k(k,h)]}{r^k},$$

where $\Upsilon(\theta) = \dfrac{u^{e\prime}(c^e(\theta))}{\mathbb{E}_\Theta[u^{e\prime}(c^e(\theta))]}$. Rearranging, we get

$$w = \mathbb{E}[\theta] f_h(k,h)(1 + \text{cov}(\Upsilon(\theta), \theta)),$$

$$r^k = \mathbb{E}[\theta] f_k(k,h)(1 + \text{cov}(\Upsilon(\theta), \theta)).$$

$\text{cov}(\Upsilon(\theta), \theta)$ is negative, and implies the same wedge across both factor markets, which must sum to factorless income $\hat{\tau}y$. It follows that $\tau = \hat{\tau} = -\text{cov}(\Upsilon(\theta), \theta)$, completing the derivation.

H.2 Derivation of Equation 13

From E5 and E6 we can write

$$\frac{c^e}{c^\gamma} = \frac{c^e_{-1}}{c^\gamma_{-1}} \frac{\beta^e}{\beta} \rho$$

We eliminate household consumption using the aggregate demand equation (E1),

$$\left(\frac{y}{c^e} - 1\right)^{-\gamma} c^{e1-\gamma} = \left(\frac{y_{-1}}{c^e_{-1}} - 1\right)^{-\gamma} c^e_{-1}{}^{1-\gamma} \frac{\beta^e}{\beta} \rho$$

Using (E2), we re-write entrepreneurial consumption in terms of leverage:

$$\left(\frac{l-(1-\beta^e)\rho}{(1-\beta^e)\rho}\right)^{-\gamma} \left(\frac{(1-\beta^e)\rho}{l}\right)^{1-\gamma} y^{1-\gamma} = \left(\frac{l_{-1}-(1-\beta^e)\rho_{-1}}{(1-\beta^e)\rho_{-1}}\right)^{-\gamma} \left(\frac{(1-\beta^e)\rho_{-1}}{l_{-1}}\right)^{1-\gamma} y_{-1}^{1-\gamma} \frac{\beta^e}{\beta} \rho$$

Rearranging yields the following relationship, linking the dynamic evolution of leverage, output, and the equity risk premium:

$$\left(\frac{l}{l_{-1}}\right)^{\gamma-1} \left(\frac{l-(1-\beta^e)\rho}{l_{-1}-(1-\beta^e)\rho_{-1}}\right)^{-\gamma} = \left(\frac{y}{y_{-1}}\right)^{\gamma-1} \frac{\rho_{-1}}{\rho_0} \tag{H1}$$

Under log utility $(\gamma = 1)$ the expression simplifies as follows:

$$l = l_{-1}\frac{\rho_0}{\rho_{-1}} + (1-\beta^e)(\rho - \rho_0)$$

Start from Equation H1

$$0 = \left(\frac{y}{y_{-1}}\right)^{\gamma-1} \frac{\rho_{-1}}{\rho_0} - \left(\frac{l}{l_{-1}}\right)^{\gamma-1} \left(\frac{l-(1-\beta^e)\rho}{l_{-1}-(1-\beta^e)\rho_{-1}}\right)^{-\gamma}$$

$$\frac{\partial \text{RHS}}{\partial y} = (\gamma-1)\frac{1}{y}\left(\frac{y}{y_{-1}}\right)^{\gamma-1} \frac{\rho_{-1}}{\rho_0}$$

$$\frac{\partial \text{RHS}}{\partial l} = \left(\gamma\frac{1}{l-(1-\beta^e)\rho} - (\gamma-1)\frac{1}{l}\right)\left(\frac{l}{l_{-1}}\right)^{\gamma-1} \left(\frac{l-(1-\beta^e)\rho}{l_{-1}-(1-\beta^e)\rho_{-1}}\right)^{-\gamma}$$

$$\frac{dl}{dy} = -\frac{l}{y}\frac{(\gamma-1)(l-(1-\beta^e)\rho)}{l+(\gamma-1)(1-\beta^e)\rho} \tag{13}$$

H.3 Derivation of Equations 11, 12

We can solve for the path of output in competitive equilibrium. Starting with the competitive equilibrium labour supply condition under log household utility:

$$h^{1-\alpha+\psi}c = \alpha z(1-\tau).$$

We can use the production function to eliminate h, and the aggregate demand equation to eliminate c, yielding

$$\left(\frac{y}{z}\right)^{\frac{1-\alpha+\psi}{\alpha}}(y-c^e) = \alpha z(1-\tau)$$

Using (E2), we can re-write the expression in terms of leverage:

$$\left(\frac{y}{z}\right)^{\frac{1-\alpha+\psi}{\alpha}}\left(y-(1-\beta^e)\frac{y\rho}{l}\right) = \alpha z(1-\tau)$$

Ultimately we derive the following relationship between output, productivity, leverage, and the equity risk premium:

$$y = z\left(\frac{\alpha l(1-\tau)}{l-(1-\beta^e)\rho}\right)^{\frac{\alpha}{1+\psi}} \tag{11}$$

Under the socially optimal wage subsidy regime, we can use Equation C2 to solve for the path of output:

$$y = z\left(\frac{\alpha l(1-\tau)\left(1+\frac{\tau}{1-\tau}\left(1-\left(\frac{l_0\rho-l\rho_0}{\rho-1}\right)\frac{(1-\beta^e)}{l_0-(1-\beta^e)\rho_0}\right)\right)}{l-(1-\beta^e)\rho}\right)^{\frac{\alpha}{1+\psi}}$$

which simplifies to

$$y = z\left(\frac{\alpha l_0}{l_0-(1-\beta^e)\rho_0}\right)^{\frac{\alpha}{1+\psi}} \tag{12}$$

I New Keynesian model and proofs

I.1 Overview of principal equations

A full derivation of the model can be found in Duncan, Mainente, and Nolan (2023). Here we provide some intuition behind the key equations in the New Keynesian version of our model.

I.1.1 The IS curve

To construct the IS curve, we start with the standard consumption Euler condition for the household sector

$$c_t = \mathbb{E}_t\left[c_{t+1}\right] - \frac{1}{\gamma} r_t$$

where $r_t := i_t - \mathbb{E}_t \pi_{t+1}$ is the real interest rate, and γ is the inverse of the intertemporal elasticity of substitution.

To write the Euler condition in terms of output, we need to incorporate the expected change in the household's consumption share of output.

$$\mathbb{E}[\Delta c_{t+1}] = \underbrace{\mathbb{E}[\Delta y_{t+1}]}_{\text{Expected change in output}} + \underbrace{\omega(1-\varphi)\mathbb{E}[\Delta l_{t+1}] - \omega\varphi\mathbb{E}[\Delta\sigma_{t+1}]}_{\text{Expected change in household's consumption share of output}}$$

Combining this condition with our consumption Euler equation, we can derive the following IS curve, which relates output to the real interest rate, the expected change in output, financial uncertainty, and leverage:

$$y_t = \mathbb{E}[y_{t+1}] - \frac{1}{\gamma} r_t + \omega(1-\varphi)\mathbb{E}_t[\Delta l_{t+1}] - \omega\varphi\mathbb{E}_t[\Delta\sigma_{t+1}], \tag{15}$$

I.1.2 The Phillips curve

Entrepreneurs produce an undifferentiated intermediate good, which in turn is sold at competitive producer prices pp_t. There exists a retail sector who purchases this intermediate good, produces a differentiated final good, and sells it to households. The retail sector is monopolistically competitive, and therefore sets a markup over the intermediate good price. The retail sector does not itself incur production costs, but is subject to Calvo-type pricing frictions.

Producer prices constitute the marginal costs of the entrepreneurial sector, inclusive of any risk premia that are increasing in production. As in the standard New Keynesian model, as output increases, household consumption increases, reducing labour supply for every wage rate, and the marginal product of labour falls. The sum of these two effects constitute the benchmark model marginal costs in Equation 18. Household consumption also moves independently of output as a result of fluctuations in leverage and uncertainty, which affect the distribution of consumption between households and entrepreneurs. This is the consumption inequality wealth effect in Equation 18. Finally, the labour wedge in Equation 18 captures the effect of increased production or increased uncertainty on the residual risk borne by entrepreneurs as a result of partial idiosyncratic risk sharing. This labour wedge term may appear as a markup over production marginal costs, but it reflects the competitively priced marginal cost of increased risk bearing.

$$\mathrm{pp}_t = \underbrace{\left(\gamma + \tfrac{\varphi+\alpha}{1-\alpha}\right) x_t - \tfrac{1+\varphi}{1-\alpha}\, z_t}_{\text{benchmark model marginal costs}} + \underbrace{\gamma\omega(1-\varphi) l_t - \gamma\omega\varphi\sigma_t}_{\text{consumption inequality wealth effect}} + \underbrace{\tau_t}_{\text{labour wedge}} - \underbrace{\varsigma_t}_{\text{wage subsidy}} . \tag{18}$$

I.1.3 The leverage curve

The leverage curve is unchanged from the flexible price model up to log-linearisation.

I.2 Proofs

Proof of Proposition 3.

We adopt the following Taylor rule

$$i_t = \phi_\pi \pi_t$$

which we substitute directly into the IS-curve in the wage subsidy policymaker's problem:

Programme 1 *The planner's problem can be expressed as follows*

$$\max_{y,l,\pi,\varsigma} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left[\begin{array}{l}(1+\Delta)\frac{\varepsilon}{\lambda}\pi_t^2+(\gamma-1+(1+\Delta)\chi)\,y_t^2-2(1+\Delta)\chi y_t z_t\\ +\Delta(\zeta-\varphi)\left((1-\varphi)l_t^2-2\varphi l_t\sigma_t\right)\\ +2\Delta(\gamma-1)\left((1-\varphi)y_t l_t-\varphi y_t\sigma_t\right)\\ +\Delta\kappa_{ll}l_t^2+2\Delta\kappa_{l\sigma}l_t\sigma_t\end{array}\right]$$
$$+\beta^t\Lambda_{1t}\left[(\zeta+\gamma-1)(y_t-y_{t+1})+\zeta(\phi_\pi\pi_t-\pi_{t+1})+(\zeta-1)\varphi l_t+\gamma\Delta\varphi(\rho_\sigma-\varphi)\sigma_t\right]$$
$$+\beta^t\Lambda_{2t}\left[\pi_t-\beta\pi_{t+1}-\lambda\left(\chi+\gamma-1\right)y_t+\lambda\chi z_t+\lambda\varsigma_t-\lambda(\zeta+\theta_l-1)l_t-\lambda(\theta_\sigma-\gamma\Delta\nu)\sigma_t\right]$$
$$+\beta^t\Lambda_{3t}\left[\zeta l_t-(\zeta-\varphi)\,l_{t-1}-\gamma\Delta\varphi\sigma_t+(1+\gamma\Delta)\varphi\sigma_{t-1}+(\gamma-1)(y_t-y_{t-1})\right]$$

where $\zeta = 1+\omega\gamma(1-\varphi)$. Under log utility ($\gamma = 1$), the first order necessary conditions of Programme 4 can be expressed as follows:

$$0 = (1+\Delta)\frac{\varepsilon}{\lambda}\pi_t+\Lambda_{1t}\zeta\phi_\pi-\frac{\Lambda_{1t-1}}{\beta}\zeta$$
$$0 = (1+\Delta)\chi y_t-(1+\Delta)\chi z_t+\zeta\Lambda_{1t}-\zeta\frac{\Lambda_{1t-1}}{\beta}$$

Combining the first order conditions above and eliminating Λ_1 yields the following optimality condition:

$$\frac{\varepsilon}{\lambda\chi}\left(\beta\pi_t-\pi_{t-1}\right)=\beta\phi_\pi\left(y_t-z_t\right)-\left(y_{t-1}-z_{t-1}\right) \quad \text{(??)}$$

Combining with the equilibrium conditions yields the following characterisation of optimal policy. First, the optimal path of inflation follows

$$\pi_t = \mu\pi_{t-1}+s_t \quad \text{(eqn:inflation-wagesubsidy-taylor)}$$

$$\varsigma_t = -\mu\left(\lambda\chi\frac{\phi_\pi-\mu}{1-\mu}+(1-\beta\mu)\right)\pi_{t-1}+u_t \quad \text{(eqn:inflation-wagesubsidy-taylor)}$$

where s_t, u_t are functions of current period shocks and leverage (which is independent of policy under log utility), increasing in the cost-push and aggregate demand components of those shocks.

■

Proof of Remark 1.

Programme 2 *The planner's problem is expressed as follows*

$$\max_{y,l,\pi,i,\varsigma} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left[\begin{array}{l}(1+\Delta)\frac{\varepsilon}{\lambda}\pi_t^2+(\gamma-1+(1+\Delta)\chi)\,y_t^2-2(1+\Delta)\chi y_t z_t\\ +\Delta(\zeta-\varphi)\left((1-\varphi)l_t^2-2\varphi l_t\sigma_t\right)\\ +2\Delta(\gamma-1)\left((1-\varphi)y_t l_t-\varphi y_t\sigma_t\right)\\ +\Delta\kappa_{ll}l_t^2+2\Delta\kappa_{l\sigma}l_t\sigma_t\end{array}\right]$$
$$+\beta^t\Lambda_{1t}\left[(\zeta+\gamma-1)(y_t-y_{t+1})+\zeta(i_t-\pi_{t+1})+(\zeta-1)\varphi l_t+\gamma\Delta\varphi(\rho_\sigma-\varphi)\sigma_t\right]$$
$$+\beta^t\Lambda_{2t}\left[\pi_t-\beta\pi_{t+1}-\lambda\left(\chi+\gamma-1\right)y_t+\chi z_t+\lambda\varsigma_t-\lambda(\zeta+\theta_l-1)l_t-\lambda(\theta_\sigma-\gamma\Delta\nu)\sigma_t\right]$$
$$+\beta^t\Lambda_{3t}\left[\zeta l_t-(\zeta-\varphi)\,l_{t-1}-\gamma\Delta\varphi\sigma_t+(1+\gamma\Delta)\varphi\sigma_{t-1}+(\gamma-1)(y_t-y_{t-1})\right]$$

From inspection of the programme above, we can see that $\pi^*=0$ (Remark 1).

■

I.3 Monetary policy only

This section solves for optimal monetary policy under commitment, in the absence of wage subsidy policy. The resulting optimality conditions are used to generate the impulse responses under *optimal monetary policy only* in Figure 8.

Programme 3 *The planner's problem is expressed as follows*

$$\max_{y,l,\pi} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left[\begin{array}{l}(1+\Delta)\frac{\varepsilon}{\lambda}\pi_t^2+(\gamma-1+(1+\Delta)\chi)\,y_t^2-2(1+\Delta)\chi y_t z_t\\ +\Delta(\zeta-\varphi)\left((1-\varphi)l_t^2-2\varphi l_t\sigma_t\right)\\ +2\Delta(\gamma-1)\left((1-\varphi)y_t l_t-\varphi y_t\sigma_t\right)\\ +\Delta\kappa_{ll}l_t^2+2\Delta\kappa_{l\sigma}l_t\sigma_t\end{array}\right]$$
$$+\beta^t\Lambda_{1t}\left[\pi_t-\beta\pi_{t+1}-\lambda\left(\chi+\gamma-1\right)y_t+\chi z_t-\lambda(\zeta+\theta_l-1)l_t-\lambda(\theta_\sigma-\gamma\Delta\nu)\sigma_t\right]$$
$$+\beta^t\Lambda_{2t}\left[\zeta l_t-(\zeta-\varphi)\,l_{t-1}-\gamma\Delta\varphi\sigma_t+(1+\gamma\Delta)\varphi\sigma_{t-1}+(\gamma-1)(y_t-y_{t-1})\right]$$

The first order conditions are

$$
\begin{aligned}
y: \quad & 0 = (\gamma - 1 + (1+\Delta)\chi)\, y_t - (1+\Delta)\chi z_t + \Delta(\gamma-1)(1-\varphi) l_t - \Delta(\gamma-1)\varphi\sigma_t \\
& \qquad - \lambda\,(\chi + \gamma - 1)\,\Lambda_{1t} + (\gamma-1)(\Lambda_{2t} - \beta\mathbb{E}_t\Lambda_{2t+1})
\end{aligned}
$$

$$
\pi: \quad 0 = (1+\Delta)\frac{\varepsilon}{\lambda}\pi_t + \Lambda_{1t} - \Lambda_{1t-1}
$$

$$
\begin{aligned}
l: \quad & 0 = \Delta(\gamma-1)(1-\varphi)y_t + \Delta[(\zeta-\varphi)(1-\varphi) + \kappa_{ll}]l_t - \Delta[(\zeta-\varphi)\varphi - \kappa_{l\sigma}]\sigma_t \\
& \qquad - \lambda(\zeta + \theta_l - 1)\Lambda_{1t} + \zeta\Lambda_{2t} - \beta(\zeta-\varphi)\mathbb{E}_t\Lambda_{2t+1}
\end{aligned}
$$

I.4 Optimal policy under discretion

We adopt the following Taylor rule

$$
i_t = \phi_\pi \pi_t
$$

which we substitute directly into the IS-curve in the wage subsidy policymaker's problem:

Programme 4 *The planner's problem is expressed as follows*

$$
\begin{aligned}
\max_{y,l,\pi,\varsigma} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{1}{2}
&\begin{bmatrix}
(1+\Delta)\frac{\varepsilon}{\lambda}\pi_t^2 + (\gamma - 1 + (1+\Delta)\chi)\, y_t^2 - 2(1+\Delta)\chi y_t z_t \\
+\Delta(\zeta-\varphi)\left((1-\varphi)l_t^2 - 2\varphi l_t\sigma_t\right) \\
+2\Delta(\gamma-1)\left((1-\varphi)y_t l_t - \varphi y_t\sigma_t\right) \\
+\Delta\kappa_{ll}l_t^2 + 2\Delta\kappa_{l\sigma}l_t\sigma_t
\end{bmatrix} \\
& + \beta^t\Lambda_{1t}\left[(\zeta+\gamma-1)(y_t - y^e) + \zeta(\phi_\pi\pi_t - \pi^e) + (\zeta-1)\varphi l_t + \gamma\Delta\varphi(\rho_\sigma - \varphi)\sigma_t\right] \\
& + \beta^t\Lambda_{2t}\left[\pi_t - \beta\pi^e - \lambda\,(\chi+\gamma-1)\,y_t + \lambda\chi z_t + \lambda\varsigma_t - \lambda(\zeta+\theta_l-1)l_t - \lambda(\theta_\sigma - \gamma\Delta\nu)\sigma_t\right] \\
& + \beta^t\Lambda_{3t}\left[l_t + n^e + (\pi_{t-1}^e - \pi_t) - y_t\right]
\end{aligned}
$$

$$
\begin{aligned}
\pi: &\quad 0 = (1+\Delta)\frac{\varepsilon}{\lambda}\pi_t + \Lambda_{1t}\zeta\phi_\pi + \Lambda_{2t} - \Lambda_{3t} \\
y: &\quad 0 = (\gamma - 1 + (1+\Delta)\chi)\, y_t - (1+\Delta)\chi z_t + \Delta(\gamma-1)\left((1-\varphi)l_t - \varphi\sigma_t\right) \\
&\qquad + (\zeta+\gamma-1)\Lambda_{1t} - \lambda\left(\chi+\gamma-1\right)\Lambda_{2t} - \Lambda_{3t} \\
l: &\quad 0 = \Delta(\zeta-\varphi)\left((1-\varphi)l_t - \varphi\sigma_t\right) + \Delta(\gamma-1)\left((1-\varphi)y_t\right) + \Delta\kappa_{ll}l_t + \Delta\kappa_{l\sigma}\sigma_t \\
&\qquad + (\zeta-1)\varphi\Lambda_{1t} - \lambda(\zeta+\theta_l-1)\Lambda_{2t} + \Lambda_{3t} \\
\varsigma: &\quad 0 = \Lambda_{2t}
\end{aligned}
$$

Let $\gamma = 1$. We can solve the above system to yield the following expression for optimal output:

$$
\begin{aligned}
y_t = z_t &+ \left(\frac{1+\mu}{\phi_\pi+\mu}\right)\frac{\varepsilon}{\chi\lambda}\pi_t \\
&- \frac{\Delta}{(1+\Delta)\chi}\left(\frac{\phi_\pi-1}{\phi_\pi+\mu}\right)\left[\left(\kappa_{ll} + (\zeta-\varphi)(1-\varphi)\right)l_t + \left(\kappa_{l\sigma} - (\zeta-\varphi)\varphi\right)\sigma_t\right]
\end{aligned} \quad \textbf{(??)}
$$

where $\mu := \left(\frac{\zeta-1}{\zeta}\right)\varphi$.

J Supplementary figures

J.1 Supplementary economic crisis figures

Figure 3 presents supplementary panels for Figure 6. The first panel shows the paths of real output in the benchmark and wage subsidy policy economies rebased such that the plots show deviations from the onset of the shock, in order to help illustrate the difference between peak-to-trough changes in output under the two rules.

The second panel shows the paths of the factor wedge τ, the wedge of inefficiency that reduces factor rental prices below their expected marginal products.

The third panel presents the ratio of the paths of marginal utility for the two groups of agents in the economy. This is the endogenous state variable in the economy, and it's persistent path reflects the risk sharing generated by the trade in securities contingent on aggregate states. 20 quarters after the onset of the crisis, while the deviation of real output from it's pre-crisis level has approximately halved, the ratio of marginal utilities is approximately at its trough. This shows that the pain of economic crises experienced by the representative worker household persists much longer than the recession itself.

J.2 Nonlinear model impulse response functions

Figures 4 and 5 present impulse responses for our nonlinear model. Both figures present a 2 standard deviation recessionary shock. For each shock and policy function, we draw 200 000 draws from the joint posterior and ergodic distributions of the model, and take the mean values of responses.

Figure 3: Economic crises with and without wage subsidy simple rules
Mean deviations from deterministic steady-state values reported
(unless otherwise stated).

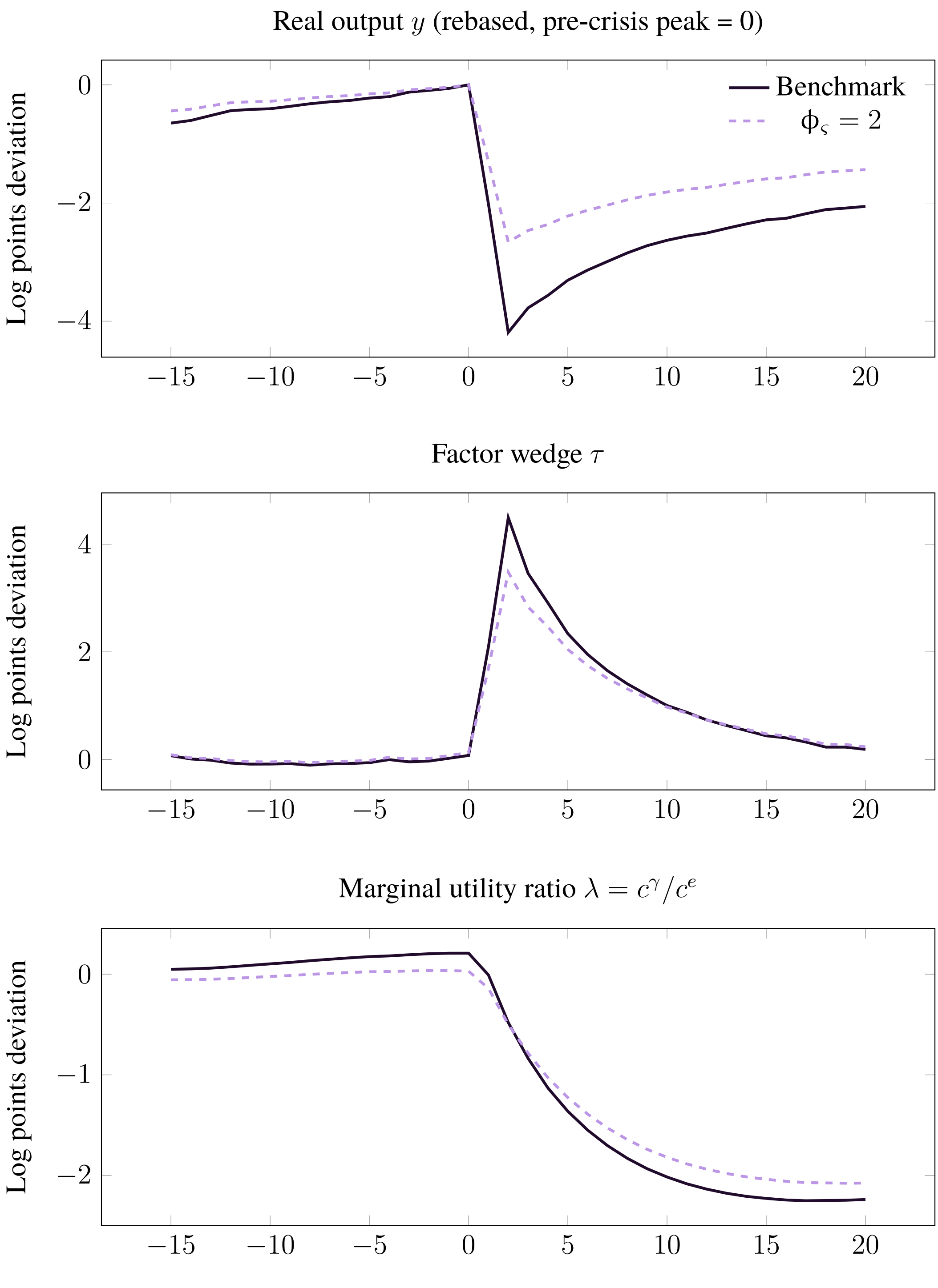
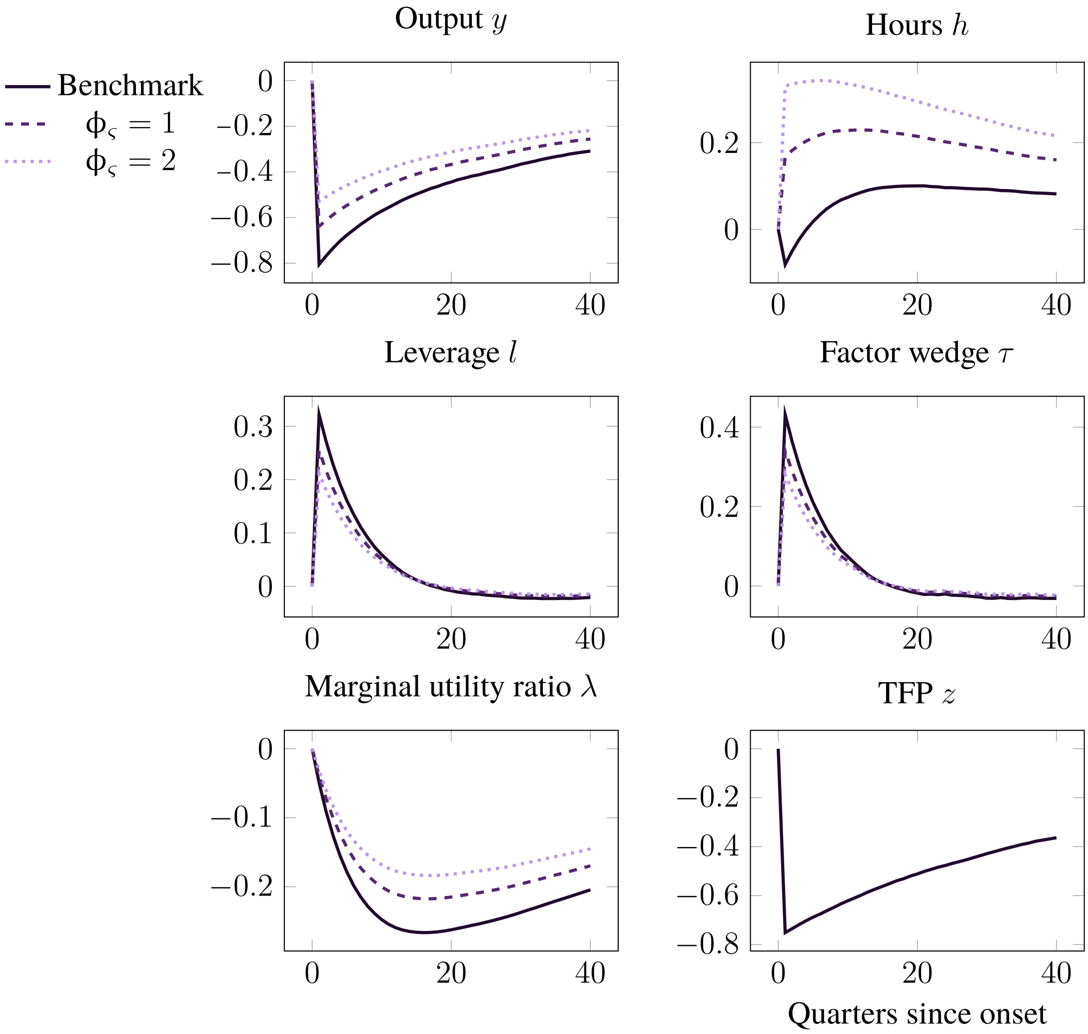


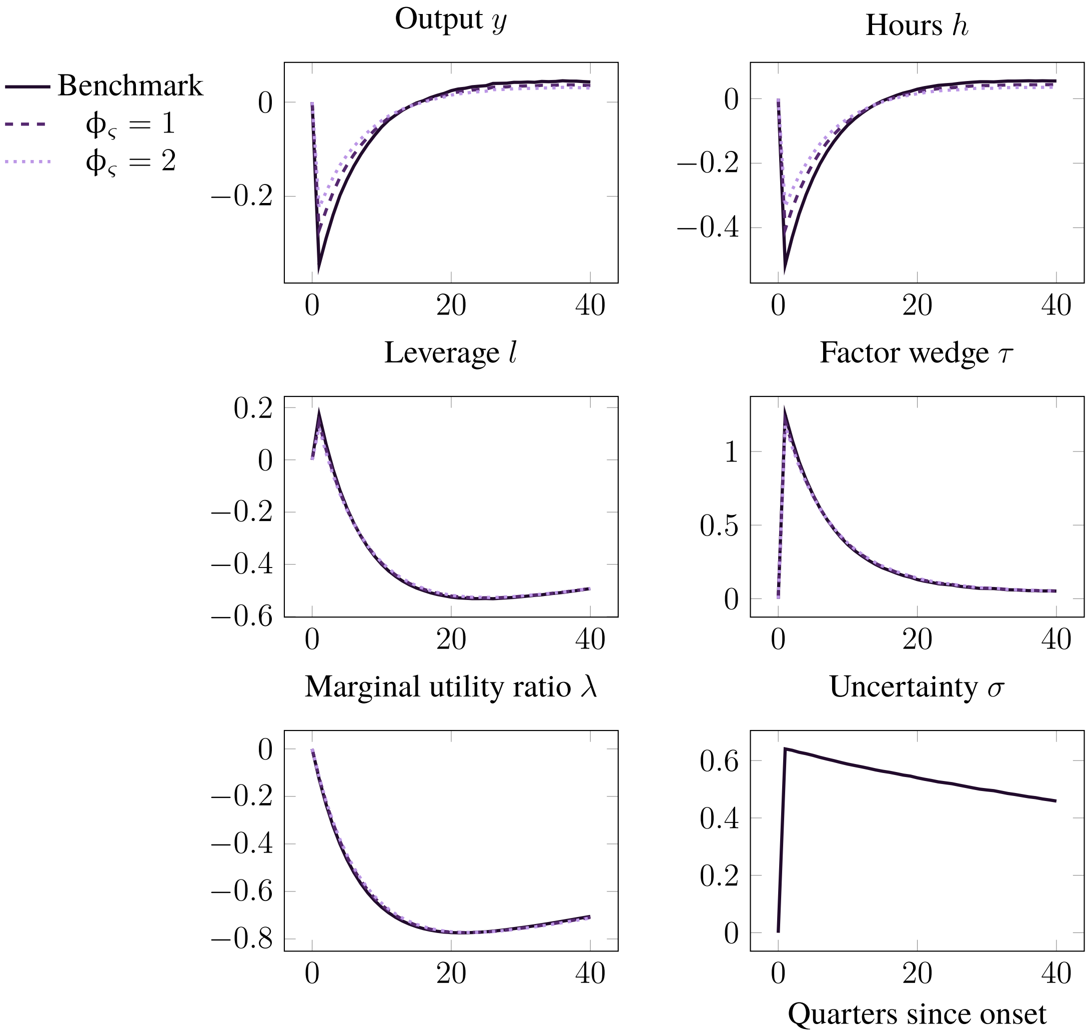
Figure 4: Impulse response functions.
2 standard deviation negative technology shock. Mean responses across posterior and initial conditions. Log points deviation from onset.



K Bank model and proofs

In this Appendix we describe a model where the financial friction lies between households and banks, who issue non-contingent deposits. Banks combine this deposit funding with equity to fund state contingent loans to firms, who have a risky production technology. There is no information asymmetry between banks and firms, but banks are matched with individual firms, and cannot diversify their

Figure 5: Impulse response functions.
2 standard deviation positive uncertainty shock. Mean responses across posterior and initial conditions. Log points deviation from onset.



loans across firms. This leaves banks with residual risk, to which they are averse. There is a risk premium charged by banks that increases when bank net worth is low, or when productive uncertainty is high.

Banks and households can allocate aggregate risk between periods, as in our benchmark model. Also, as in our benchmark model, the high risk associated with high returns in downturns discourages banks from insuring their balance sheets against recessionary risks, ultimately leading to countercyclical levarage, and tightened lending conditions in downturns, and a macroprudential externality.

In the passages below, we show a countercyclical wage subsidy can increase welfare in this environment, despite the absence of a labour wedge in the competitive equilibrium allocations in the model, and despite the fact that banks do not directly hire labour. Banks still benefit from the pecuniary externality of the wage subsidy, which increases the marginal value of bank equity in downturns, and ex ante encourages banks to adjust their risk profile to ensure that they have additional equity in periods when the wage subsidy is positive.

K.1 Banks and productive firms

Each banker, superscript b, starts with net worth n^b, and combines this with loans from the household to fund an investment in a single productive firm with idiosyncratic productivity θ.

Productive firms, owned by households, fund purchases of capital k with loans from banks. They then draw productivity draw θ, which is drawn from a distribution with $\mathbb{E}[\theta] = 1$. Upon realisation of θ, the productive firm hires employees, equating their marginal product with the market wage rate

$$w = \theta\alpha \left(\frac{k}{h}\right)^{1-\alpha}.$$

Firm specific labour demand is

$$h(\theta) = \left(\frac{\theta\alpha}{w}\right)^{\frac{1}{1-\alpha}}.$$

We can derive aggregate output and labour demand as follows:

$$\begin{aligned} y &= \int \theta h(\theta)^{\alpha}\, dG(\theta) \\ &= \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} \int \theta^{\frac{1}{1-\alpha}}\, dG(\theta) \end{aligned}$$

$$\bar{h} = \int h(\theta)\, dG(\theta)$$
$$= \left(\frac{\alpha}{w}\right)^{\frac{1}{1-\alpha}} \int \theta^{\frac{1}{1-\alpha}}\, dG(\theta)$$

Combining yields the following aggregate production function

$$y = h^{\alpha} \left[\int \theta^{\frac{1}{1-\alpha}} dG(\theta)\right]^{1-\alpha}$$

Productive firms are competitive in factor and product markets, and their matched bank can observe their true productivity without cost. In equilibrium, the payment disbursed from a firm with productivity draw θ to their matched bank is

$$bR^k(\theta) = (1-\alpha)\theta h(\theta)^{\alpha}$$

$$bR^k(\theta) = (1-\alpha)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} \theta^{\frac{1}{1-\alpha}}$$

where b denotes the amount of funds lent to the firm by their matched bank.

Ultimately, the risk of fluctuations in θ is borne by the matched bank, who receives a high loan repayment when θ is high, and a low loan repayment when θ is low. Loan repayments to banks are not observable to depositors, and depositors don't have access to any audit technology. Banks offer a non-contingent deposit product, with bankers accepting the residual risk.

The opportunity cost of risky loans to firms is the risk free rate,

$$\mathbb{E}_{\theta} R^k(\theta) u^{b\prime}(c^b(\theta)) = (1+r)\mathbb{E}_{\theta} u^{b\prime}(c^b(\theta)).$$

It follows that the equity risk premium is

$$\rho = 1 - \text{cov}_{\theta}\left(\frac{R^k(\theta)}{1+r}, \frac{u^{b\prime}(c^b(\theta))}{\mathbb{E}_{\theta} u^{b\prime}(c^b(\theta))}\right).$$

Note that

$$c^b(\theta) = (1-\beta^b)\left[b\left(R^k(\theta) - (1+r)\right) + n^b\right].$$

When the bank's net worth n^b is low, or the volatility of θ is high, individual bankers' consumption is more sensitive to realised productivity outturns, and the equity risk premium increases. Ultimately this generates a tightening of lending conditions and a decrease in output during periods of high uncertainty, or low bank net worth.

Banks enjoy consumption with log utility and their intertemporal problem is isomorphic to that of the entrepreneurs described in Subsection 1.1.

K.2 Households

Worker households enjoy consumption with log utility, and dislike labour effort with constant inverse Frisch elasticity ψ.

$$u(c,h) = \log c - \frac{h^{1+\psi}}{1+\psi}.$$

Households' intertemporal problem is unchanged from that described in Subsection 1.2.

K.3 Equilibrium

From the above, the following system of equations determined competitive equilibrium allocations:

$$h^\psi = \frac{w}{c} \qquad \text{(labour supply)}$$

$$y = c + c^b \qquad \text{(aggregate demand)}$$

$$\alpha y = wh \qquad \text{(labour demand)}$$

$$y = \hat{\theta} h^\alpha \qquad \text{(production)}$$

where

$$\hat{\theta} := \left[\int \theta^{\frac{1}{1-\alpha}} dG(\theta)\right]^{1-\alpha}$$

Let $\Delta^b := \frac{c^b}{y}$. From the equations above we can solve for equilibrium hours

$$h = \left(\frac{\alpha}{1-\Delta^b}\right)^{\frac{1}{1+\psi}}$$

and output

$$y = \hat{\theta}\left(\frac{\alpha}{1-\Delta^b}\right)^{\frac{\alpha}{1+\psi}},$$

or

$$y = \hat{\theta}\left(\frac{\alpha l^b}{l^b-(1-\beta^b)\rho}\right)^{\frac{\alpha}{1+\psi}} \tag{K1}$$

for appropriately defined risk-adjusted leverage measure $l^b := \frac{y}{n^b}$. Note that in this bank model, it is easier than in our benchmark model to shift between risk adjusted leverage l^b and balance-sheet measures of leverage:

$$l^b = \frac{\rho}{1-\alpha}\frac{b}{q^b},$$

where $\frac{b}{q^b}$ is the ratio of the face value of bank loans to bank net worth (equity).

Equation K1 is very similar to Equation 11, which described the competitive equilibrium path of output in our benchmark model given the same parameter restrictions on households. The main difference between the two is that in the bank model, the labour wedge no longer remains, as productive firms now hire labour after observing their realised productivity and therefore do not mark down wages by a risk premium. There is also an efficiency wedge $\hat{\theta}$ in the bank model that derives from the misallocation of capital, arising from the fact that firms rent capital before realising their productivity.

The steps of Proposition 1 also follow directly in the bank model. The path of output under optimal wage subsidies follows

$$y = \hat{\theta}\left(\frac{\alpha l_0^b}{l_0^b-(1-\beta^b)\rho_0}\right)^{\frac{\alpha}{1+\psi}} \tag{K2}$$

Again, Equation K2 is very similar to Equation 12, which described the path of output under optimal wage subsidies for our benchmark model. Comparing Equations K1 and K2, we can see that the optimal wage subsidy increases in bank leverage.

In contrast with our benchmark model, in our bank model the steady state optimal wage subsidy is zero.

While banks don't hire labour in this model, the marginal value of bank net worth is still increasing in labour supply. The capital goods owned and intermediated by banks are complements to labour. Wage subsidies increase labour supply and banks anticipating future wage subsidies will adjust their risk taking to ensure that they have more equity in periods with a positive wage subsidy, all else equal.

K.4 Closing the model

To close the model, we need to specify a distribution for θ. Perhaps the simplest approach is to set

$$\theta = \begin{cases} \bar{\theta} & \text{with probability } \pi, \\ 0 & \text{otherwise.} \end{cases}$$

Firms who receive low draws have zero productivity. We interpret this as default. In this case, $\hat{\theta}$ becomes $\pi^{1-\alpha}\bar{\theta}$. By allowing variation in both π and $\bar{\theta}$, it remains possible to capture fluctuations in uncertainty and aggregate total factor productivity. From the fundamental theorem of asset pricing,

$$1 = \mathbb{E}\left[\left(\frac{R^k(\theta)}{1+r}\right)\left(\frac{u^{b\prime}(c^b(\theta))}{\mathbb{E}_\theta u^{b\prime}(c^b(\theta))}\right)\right],$$

and the banker's optimal consumption plan,

$$c^b(\theta) = (1-\beta^b)q^b\left[\frac{b}{q^b}R^k(\theta) + \left(1-\frac{b}{q^b}\right)(1+r)\right],$$

we can solve for banker leverage and returns:

$$l^b = \frac{\rho}{1-\alpha}\frac{\pi(\rho-1)}{\rho-\pi}. \tag{K3}$$

Note that Equation K3 does not contain $\bar{\theta}$ which acts as a productivity shifter in this model. It is best to think of a decrease in the probability of project success π (or an increase in the probability of default $1-\pi$) as representing an uncertainty shock in this model, and it is clear from Equation K3 that such a shock would increase the equity risk premium for every level of leverage, similar to an uncertainty shock in our benchmark model. From the equilibrium in intertemporal risk sharing (see Equation 3) we have

$$\frac{c^b}{c} = \frac{\rho}{\rho_0}\frac{c^b_{-1}}{c_{-1}}$$

which can be solved for current and previous period leverage and equity risk premia:

$$l^b = \frac{\rho_0}{\rho_{-1}} l^b_{-1} + (1 - \beta^b)(\rho - \rho_0) \tag{K4}$$

Combining Equations K1, K3, and K4 yields a three equation dynamic model.

References

Alfred Duncan. Reverse mode differentiation for DSGE models. Studies in Economics 2108, School of Economics, University of Kent, July 2021.

Alfred Duncan, João Pedro De Camargo Mainente, and Charles Nolan. Optimal Macro-Financial Policies in a New Keynesian Model with Privately Optimal Risk Taking, 2023.

Hong Ge, Kai Xu, and Zoubin Ghahramani. Turing: a language for flexible probabilistic inference. In *International Conference on Artificial Intelligence and Statistics, AISTATS 2018, 9-11 April 2018, Playa Blanca, Lanzarote, Canary Islands, Spain*, pages 1682–1690, 2018. URL `http://proceedings.mlr.press/v84/ge18b.html`.

Matthew D. Hoffman and Andrew Gelman. The no-u-turn sampler: Adaptively setting path lengths in hamiltonian monte carlo, 2011.