Factor Investing and Currency Portfolio Management

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Abstract

Currency-specific pricing factors are pervasive in international asset pricing. However, portfolio and risk managements based on currency factors, instead of individual currencies, are rarely discussed. This paper tries to fill this gap by modelling dynamic correlations and non-normality among currency factors. By considering the four most popular currency factors: the dollar risk factor, the carry trade factor, the currency momentum factor and the currency value factor, we find that a dynamic conditional correlation copula (DCC-copula) model with skewed-t kernel fits the joint distribution well. For a risk-averse investor, attractive economic value is added by the DCC-copula model in currency factor investing, while ignoring the correlation structure or assuming naive distributions (such as joint normal distribution) brings significant costs.

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1 Introduction

Currency anomalies are difficult to fit into a stochastic discount factor (SDF) model with traditional risk factors (e.g., Burnside et al. [2010], Burnside [2011] and Lustig et al. [2011]) which has led researchers to construct currency market-specific pricing factors.

Lustig et al. [2011] propose the dollar risk factor (DOL) and the carry trade factor (HML). The DOL factor is the cross-sectional average of all currency excess returns. The HML factor is the return of high interest rate currencies minus the return of low interest rate currencies. Menkhoff et al. [2012a] propose the currency volatility factor which is the cross-sectional average of volatility innovations (volatility factor) of all currencies. Della Corte et al. [2016] and Della Corte et al. [2021] introduce the currency volatility risk premia. They find currencies that are cheap to insure (by using currency options) provide higher returns. Asness et al. [2013], Menkhoff et al. [2017] and Kroencke et al. [2014] discuss the currency value strategy (VAL) which is the return difference between over-valued currencies and under-valued currencies. Whether the currency is over- or under-valued depends on the consumer price index (CPI) in a country other than the US. Menkhoff et al. [2012b] find that the excess return of currency momentum strategies (MOM), in cross-section, is impressive. Burnside et al. [2011] find that the currency momentum is not correlated with other currency factors. Among others, the forex factors cited above have became pervasive in the literature. An SDF model that employs a DOL and another currency-specific factor capture substantial cross-sectional carry trade returns.

Factor investing has been widely studied in the equity market. It involves using the "factors"[1] instead of individual assets, as the basic unit in portfolio constructions and risk managements. Given that a composite set of currency factors has been established, a straightforward question arises: How should investors choose between currency factors in forming portfolios, or put differently, what is the economic value of the factor when investing in the currency market? Surprisingly little attention has been paid to this research question.

We try to fill this gap focusing of the four most popular currency factors, namely, DOL, HML, VAL, and MOM. Using factors, instead of individual currencies, as the basic unit in forming optimal currency portfolios provides two advantages. First, the country-specific risk can be averaged out. Second, factors are rebalanced every month so there is stable risk property through time, whereas the risk property of individual currencies could change with the economic fundamentals or government policies.

[1] Such as the market, value, size, and momentum factors in the equity market

Modelling the correlation structure is of great importance in factor investing, especially for currency factors. Unlike equity factors which are nearly orthogonal to each other, we find the currency factors are correlated with each other. Menkhoff et al. [2012a] show that the mimicking portfolio of the currency volatility factor loads in a similar way as to a carry trade strategy. The HML factor and VAL factor could also be correlated. Because of CPI and interest rates, the sorting variables of HML and VAL, are highly correlated with each other. We also show that the correlation has a time-variant property. Thus, the dynamic conditional correlation (DCC) model of Engle [2002] is used.

We follow Christoffersen and Langlois [2013] and Arnott et al. [2019] who suggest investors should not ignore tail risk and joint non-normality in factor investing. In fact, currency carry trades and momemtums carry negative skew and excess kurtosis (See, for example, Brunnermeier et al. 2008 and Menkhoff et al. 2012b). We report strong evidence of joint non-normality between currency factors. We employ a battery of copula models to model the joint distribution of currency factors.

In this paper, the dynamic conditional correlation copula (DCC-copula) model with normal, student t, and skewed t kernels are employed. We show that there is non-linear correlation structures among currency factors. Thus, the DCC-copula with skewed t kernel fits the data best in terms of the log-likelihood.

Based on the DCC-copula model, we build optimal currency portfolios with 24 years of weekly out-of-sample returns. Under the setting of a constant relative risk aversion (CRRA) utility investor, we find the significant economic value of the model in forming optimal currency portfolios. We consider two benchmark models: i) the orthogonal model which ignores the correlation structure ii) the normal model which assumes linear correlation. The DCC-copula with skewed t kernel outperforms two benchmark models in terms of Sharp ratios and certainty equivalents. This result is robust across different levels of risk aversion, the sub-sample of developed or developing currencies and even stronger when transaction costs are considered.

The final part of this paper focuses on risk management. We forecast the value-at-risk (Var) and expected shortfalls (ES) of individual factors. The DCC-copula model still shows the robustness compared with the benchmark models. We apply the Diebold-Mariano tests, following Patton et al. [2019], to rank the performance of the models. The DCC-copula with skewed t rank the best. Modelling the non-normality and the dynamic correlation improve the ability to forecast the risk measures.

As far as we are aware, this is one of the few papers to design optimal currency portfolios using forex factors and to investigate the correlation structure and non-normality between currency factors. Previous literature is limited in that it focuses on individual currency. For example, Patton [2006] first introduces the copula model to discuss tail dependence for mark-dollar and yen-dollar exchange rates. Bouyé and Salmon [2009] derive the implicit form of conditional quantile relations of dollar-yen, dollar-sterling and dollar-DM. One of the few closely related studies is Barroso and Santa-Clara [2015] who form optimal currency portfolios and detect relevant variables by using the portfolio policies method [Brandt et al., 2009]. They show that carry, momentum and value work better than fundamentals on designing optimal portfolios. Our paper extends Barroso and Santa-Clara [2015] as we provide a detailed analysis of factor correlations and introduce non-linearity.

As we mentioned, our paper is also related to the large literature in the equity arena focusing on factor investing. Christoffersen and Langlois [2013] apply the copula model to the market factor, size factor, value factor and momentum factor in the out-of-sample data set and show that correlations of the factors in the equity market are not orthogonal. Arnott et al. [2019] also discuss the correlations between the factors in the equity market.

Our paper is also related to the literature using copula models to manage tail behavior in the joint distributions of financial time series. Patton [2006] uses normal copula and student-t copula to model the bivariate distribution of individual currencies. Patton [2006] shows that, compared with the normal copula, the student t can handle the kurtosis. Christoffersen et al. [2012] propose the constant and dynamic copula models to focus on the multivariate joint distribution. The skewed t copula model proposed by Christoffersen et al. [2012] shows that accounting for asymmetry is also important. Furthermore, the dynamic conditional correlation copula model can also handle the time-varying changes of the correlations between the variables.

The paper is organized as follows: section 2 describes our data and relevant statistic description; section 3 presents asymmetry tail dependence modelling of forex factors; section 4 introduces the economic implications of the copula model by constructing optimal portfolios for risk-averse investors; section 5 shows the application of the copula in risk management; section 6 provides the conclusion.

2 Data and Currency Factors

2.1 Data

We use weekly forward and spot rates and price level of consumer goods from January 1, 1989, to March 20, 2020, for 31 active trading currencies.[2] The data are all from DATASTREAM.

2.2 Currency Factors

The excess return of carry trade is calculated using term t log forward rate less term $t+1$ log spot rate for each currency.

$$ER_{j,t} = f_{j,t} - s_{j,t+1}$$

Where the $f_{j,t}$ denotes the term t log forward rate of currency j. The $s_{j,t+1}$ denotes the term $t+1$ log spot rate of currency j. $ER_{j,t}$ is the excess return at term t of currency j.

The dollar risk factor (DOL) is simply the mean of 31 currencies' excess return.

$$DOL_t = mean\,(ER_{j,t})$$

In constructing *the high minus low carry trade factor* (HML), we follow Lustig et al. [2014] and sort the currency returns from lowest to highest based on the forward premium and allocate them into five portfolios. The HML factor is the difference between the mean returns of the fifth portfolio (the largest forward premium) and the first portfolio (the smallest forward premium). We denote it as the carry trade factor.

$$HML_t = Port_{H,t} - Port_{L,t}$$

[2] List of currencies: 10 important developed countries' currencies (AUDUSD, CADUSD, CHFUSD, DKKUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD and SEKUSD); 8 important emerging countries' currencies (CZKUSD, HUFUSD, ILSUSD, ISKUSD, PLNUSD, RUBUSD, TRYUSD and ZARUSD); 6 Asian currencies (HKDUSD, KRWUSD, MYRUSD, PHPUSD, SGDUSD and THBUSD); 5 Latin American currencies (BRLUSD, CLPUSD, COPUSD, MXNUSD and PENUSD); 2 Middle East currencies (JODUSD and KWDUSD). Note that the developed countries' dataset, which we apply in the later section, just includes the 10 important developed countries. The developing countries' dataset include the rest of 21 countries' currencies.

Where $Port_{H,t} = mean\left(ER_{j,t,largest\,forward\,premium}\right)$ and $Port_{L,t} = mean\left(ER_{j,t,smallest\,forward\,premium}\right)$.

For *the currency momentum* (MOM) factor, we follow Menkhoff et al. [2012b] and use the previous 6-week formation period and 1-week holding period to sort the currencies into five portfolios based on their lagged returns. The MOM factor is the difference between the mean returns of the lowest lagged return portfolio and the highest lagged return portfolio.

$$MOM_t = Port_{HM,t} - Port_{LM,t}$$

Where $Port_{HM,t} = mean\left(ER_{j,t,highest\,lagged\,return}\right)$ and $Port_{LM,t} = mean\left(ER_{j,t,lowest\,lagged\,return}\right)$.

We follow Kroencke et al. [2014] to construct *the currency value factor* (VAL) factor. For currency j, we first determine the real exchange rate $Q_{j,t}$ at time t:

$$Q_{j,t} = \frac{S_{j,t}P_{j,t}}{P^*_{j,t}}$$

where $P_{j,t}$ denotes the price level of consumer goods in country j at term t; $P^*_{j,t}$ the corresponding foreign price level (here USD); $S_{j,t}$ is the spot exchange rate.

$$F_{VAL,j,t} = \left(\frac{Q_{j,t-3}}{Q_{j,t-13}} - 1\right)(-1)$$

The VAL factor can be calculated as the above equation by the real exchange rate with 3 and 13 weeks. Since we want the factor portfolio returns, we then sort the currency returns from lowest to highest based on the VAL factor and allocate them into five portfolios to obtain the VAL portfolio as follows:

$$VAL_t = Port_{HV,t} - Port_{LV,t}$$

Where $Port_{HM,t} = mean\left(ER_{j,t,highest\,VAL\,factor}\right)$ and $Port_{LM,t} = mean\left(ER_{j,t,lowest\,VAL\,factor}\right)$.

2.3 Descriptive statistics

In Table 1 we report descriptive statistics for 4 currency factors. The table shows annualized mean returns, Newey-West t-statistics, standard deviations, skewness, kurtosis, autocorrelation coefficient and linear correlation matrix. The annualized mean return is the highest for the carry trade factor HML and is negative for the DOL factor. All factors show excess kurtosis. The skewness is negative for most factors but positive for VAL. The second panel shows the auto-correlation coefficients. Most of the factors, apart from the DOL, have strong second-order and third-order auto-correlation.

We report the sample linear correlation matrix in the last panel. There are significant pairs of correlations among all factors. We observe a negative correlation between MOM and DOL which is consistent with Daniel and Moskowitz [2016]. Surprisingly, we find that correlation between HML and MOM is positive and HML and VAL is negative. Since most studies have reported a negative correlation between HML and MOM factors (see, for example, Burnside et al. 2011), we investigate this issue further by splitting the full sample into two: one including the 2008 financial crisis and one not including it. Table 2 shows the results. The financial crisis does not seem to be driving that result (see Table 1). However, when we split the sample into developed and developing countries, we find a clear difference for correlations of HML and MOM or HML and VAL. In developed countries, the factors HML and MOM have the expected negative correlation while HML and VAL are positively correlated. For developing countries, HML and VAL are also positively correlated.

[Table 1 factor descriptive stats table is about here]

The empirical evidence above, although in a simple form, does support our view: correlation among forex factors is not captured by a normal distribution.

3 Modelling Asymmetry Between Currency Factors

In this section, we model asymmetry between currency factors. First, we apply the threshold correlation to test the non-linear correlation between factors. We apply the AR-GARCH model to mitigate auto-correlation (evidence from table 1) and we introduce univariate volatility focusing on the joint distribution between factors.[3] Finally, we introduce the copula model and present the results of the asymmetry.

[3]The AR-GARCH model and the results are presented in the Appendix A

3.1 Modelling Dependence Between Currency Factors

In this section, we present a detailed analysis of the dependence structure between forex factors. We model the dependence structure for each pair of currency factors using threshold correlations or quantile dependence, as in Christoffersen and Langlois [2013]. [4] The idea here is to characterize the dependence of two variables in the joint lower or joint upper tails, respectively. Unlike linear correlation, this approach involves modelling the asymmetric dependence structure between extreme events, which is appropriate in the presence of skewness and excess kurtosis. We define the threshold correlation $\overline{\rho}_{i,j}(u)$ for any two factors i and j as follows:

$$\overline{\rho}_{i,j}(u) = \begin{cases} corr(r_i, r_j | r_i < F_i^{-1}(u), r_j < F_j^{-1}(u)) & when\, u \leq 0.5 \\ corr(r_i, r_j | r_i \geq F_i^{-1}(u), r_j \geq F_j^{-1}(u)) & when\, u \geq 0.5 \end{cases}$$

Where u is a threshold between 0 and 1, and $F_i^{-1}(u)$ is the empirical quantile function of the univariate distribution of r_i.

Figure 2 plots the empirical threshold correlation against the threshold u for the each pair of factors.[5] As a comparison, we assume that the theoretical threshold correlation, given the factors pairs, follows a bivariate normal distribution (see the dashed line). For bivariate normal distributions, the threshold correlation will be symmetric around 0.5 and will gradually approach 0. Figure 2 shows that the bivariate normal assumption does not hold, as we observe increasing correlations in extreme events. The empirical correlations show a significant degree of asymmetry, especially in the tail. Correlations between factors appear to be, in general, positive and large.[6] Our results show that assuming linear dependency between factors will underestimate portfolio risk in extreme event scenarios, and so diversification, in this case, will not work in reducing the overall risk exposure. The empirical results above are important and new as they shed new light on the literature (see for example Kroencke et al. 2014; Brandt et al. 2009) and show that dependence between forex factors is very significant.

[Figure 2 Threshold correlation graph about here]

[4] The same method was used by Longin and Solnik [2001], Ang and Chen [2002], Ang and Bekaert [2002] and Patton [2004]

[5] We follow Christoffersen and Langlois [2013] who compute the threshold correlation when at least 20 pairs of values are available.

[6] Although these results appear rather interesting and worthy of further investigation, this is not the objective of this paper and we leave this question for future research

3.2 Copula Models

To model non-normality, we use copula models as in Patton [2006]. Before modelling tail dependence, we apply the univariate autoregressive-non-linear generalized autoregressive conditional heteroscedasiticity (AR-NGARCH) model to get the residuals for the factors.[7] We then use copula as it is a flexible framework to characterize multivariate distributions. The joint probability density function $f_t(r_{1,t+1}, ..., r_{N,t+1})$ of the N forex pricing factors can be decomposed as follows:

$$f_t(r_{1,t+1}, ..., r_{N,t+1}) = c_t(\eta_{1,t+1}, ..., \eta_{N,t+1}) \prod_{j=1}^{N} f_{j,t}(r_{j,t+1}),$$

Where $f_{j,t}(r_{j,t+1})$ is the univariate marginal probability density function for factor j and time t; $c_t(\eta_{1,t+1}, ..., \eta_{N,t+1})$ is the conditional density copula function;$\eta_{j,t+1}$ is the marginal probability density for factor j.

$$\eta_{j,t+1} = F_{j,t}(r_{j,t+1}) \equiv \int_{-\infty}^{r_{j,t+1}} f_{j,t}(r)dr$$

The $F_{j,t}$ is the cumulative distribution function (CDF) of the skewed t distribution of Hansen [1994].

The most common functional forms of copula models in financial time series are the normal copula and the student t copula. However, these two copula models can only generate symmetric multivariate distributions and fail to account for the asymmetry in threshold correlations that we have empirically shown above for the factors. Copulas from the Archimedean family (The Clayton, the Gumbel and Joe-Clayton specifications) can be used for asymmetric bivariate distributions, but they are not easily generalized to high dimensional cases.

Demarta and McNeil [2005] propose the skewed t distribution and the skewed t copula which have been widely used in financial modelling.[8] The skewed t distribution belongs to the

[7] The detailed model and results of the univaraite model are shown in Appendix A

[8] The skewed t copula is used by Christoffersen et al. [2012] for the analysis of international equity diversification and Christoffersen and Langlois [2013] for equity market factor modelling. Cerrato et al. [2017a] use this model for joint credit risk analysis of UK banks. Cerrato et al. [2017b] model the higher-order components of equity portfolios.

multivariate normal variance mixtures class. An N-dimensional skewed t random variable X has the following representation:

$$X = \sqrt{W}Z + \lambda W \tag{1}$$

Where W follows an inverse Gamma $IG(v/2, v/2)$ distribution; Z is a N-dimensional normal distribution with mean 0 and correlation matrix Ψ; λ is a $N \times 1$ asymmetry parameter vector. The multivariate probability density function of the skewed t distribution is:

$$f_t(r; v, \lambda, \Psi) = \frac{2^{\frac{2-(v+N)}{2}} K_{\frac{v+N}{2}}\left(\sqrt{(v + z^{*\top}\mathit{\Psi}^{-1}z^*)\lambda^\top\mathit{\Psi}^{-1}\lambda}\right) e^{z^{*\top}\mathit{\Psi}^{-1}\lambda}}{\Gamma\left(\frac{v}{2}\right)(\pi v)^{\frac{N}{2}} \mid \mathit{\Psi} \mid^{\frac{1}{2}} \left(\sqrt{(v + z^{*\top}\mathit{\Psi}^{-1}z^*)\lambda^\top\mathit{\Psi}^{-1}\lambda}\right)^{-\frac{v+N}{2}} \left(1 + \frac{z^{*\top}\mathit{\Psi}^{-1}z^*}{v}\right)^{\frac{v+N}{2}}}$$

The copula density function derived from the above probability density function is:

$$\begin{aligned} c_t(\eta; \lambda, v, \Psi) \quad &= \quad \frac{2^{\frac{(v-2)(N-1)}{2}} K_{\frac{v+N}{2}}\left(\sqrt{(v + z^{*\top}\mathit{\Psi}^{-1}z^*)\lambda^\top\mathit{\Psi}^{-1}\lambda}\right) e^{z^{*\top}\mathit{\Psi}^{-1}\lambda}}{\Gamma\left(\frac{v}{2}\right)^{1-N} \mid \mathit{\Psi} \mid^{\frac{1}{2}} \left(\sqrt{(v + z^{*\top}\mathit{\Psi}^{-1}z^*)\lambda^\top\mathit{\Psi}^{-1}\lambda}\right)^{-\frac{v+N}{2}} \left(1 + \frac{z^{*\top}\mathit{\Psi}^{-1}z^*}{v}\right)^{\frac{v+N}{2}}} \\ &\times \prod_{j=1}^{N} \frac{\left(\sqrt{\left(v + \left(z_j^*\right)^2\right)\lambda_j^2}\right)^{-\frac{v+1}{2}} \left(1 + \frac{\left(z_j^*\right)^2}{v}\right)^{\frac{v+1}{2}}}{K_{\frac{v+1}{2}}\left(\sqrt{\left(v + \left(z_j^*\right)^2\right)\lambda_j^2}\right) e^{z_j^*\lambda_j}} \end{aligned}$$

Where K$(\cdot)$ denotes the modified Bessel function of the second kind, and $z^* = t_{\lambda,v}^{-1}(\eta_i)$ denotes the copula shocks where $t_{\lambda,v}(\eta_i)$ is the univariate skewed t distribution:

$$t_{\lambda,v}(\eta_i) = \int_{-\infty}^{\eta_i} \frac{2^{1-\frac{v+1}{2}} K_{\frac{v+1}{2}}\left(\sqrt{(v + x^2)\lambda_i^2}\right) e^{x\lambda_i}}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi v}\left(\sqrt{(v + x^2)\lambda_i^2}\right)^{-\frac{v+1}{2}} \left(1 + \frac{x^2}{v}\right)^{\frac{v+1}{2}}} dx$$

However, a closed-form solution for skewed t quantile function is not available. We use simulation to define the quantile function and employ 1,000,000 replications of equation 1.

3.3 Modelling Dynamic Dependence Between forex Factors

Another interesting feature of the results above is that correlations change over time. We account for this feature following Engle [2002] and use a dynamic conditional correlation (DCC) model, where the correlation matrix dynamic is generated as 2

$$Q_t = Q(1 - \beta_c - \alpha_c) + \beta_c Q_{t-1} + \alpha_c z_{t-1} z_{t-t}^T \tag{2}$$

In the case of N pricing factors, $Q_t$is a $N \times N$ positive semi-definite matrix for time t;$\alpha_c$and β_c are scalars; $z_t$is a $N \times 1$ row vector of standardized residuals with jth entry $z_{j,t} = F_c^{-1}(\eta_{j,t})$, where F_c^{-1} is the inverse CDF from copula estimation; Q is a constant matrix which is a full-sample correlation matrix. The dynamic conditional correlation between factor i and j for time t is defined as

$$\Psi_{ij,t} = \frac{Q_{ij,t}}{\sqrt{Q_{ii,t} Q_{jj,t}}}$$

Coefficient $\beta_c$and α_c are estimated to allow the dynamic correlation. Note that the dynamic copula mean-reverts to the full sample correlation matrix Q. The estimates of coefficient $\beta_c$and α_c are shown in Table 3.

3.4 Estimation Method

We use a composite log-likelihood estimation introduced by Engle et al. [2009] and Christoffersen et al. [2012].[9] The composite likelihood function in our case is defined as :

$$CL(\theta) = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j>i} \ln c_t \left(\eta_{i,t}, \eta_{j,t}; \theta_{i,j}\right)$$

Where θ is the parameter set; $c_t(\eta_{i,t}, \eta_{j,t}; \theta_{i,j})$ is the bivariate copula distribution of factor pair i and j. We maximize the composite log-likelihood function $CL(\theta)$ to get the Copula coefficient estimates $\theta_{i,j}$ for each factor pair. We then average $\theta_{i,j}$to obtain an estimator of the

[9] Engle et al. [2009] find that in the large-scale DCC model, the traditional likelihood method yields biased estimates.

[Table 3 Copula results about here]

[Figure 3 Dynamic correlations of residuals about here]

parameter set θ. The standard errors are based on Engle et al. [2009]. Following Christoffersen et al. [2012], all the copula models are estimated by this method. In the Appendix, we also report the parameter estimates from maximizing the conventional likelihood function along with parameter standard error based on Chen and Fan [2006].

3.5 Empirical Results

The first panel of Table 3 shows the composite likelihood estimates for constant/dynamic parameters of normal, student t and skewed t copula. The degree of freedom ν and most of skewness parameter λ in skewed t copula are all significant. This is consistent with non-normal and asymmetric dependence between currency factors. For the constant copula models where the constant correlation structure is assumed, the full sample correlation estimates are reported. For dynamic copula models, we report DCC parameter estimates $\alpha_c, \beta_c$and long-term mean-reverting correlation matrix Q as in equation 2. The estimates of Q are about the same as for the full sample correlation of the static copula models. DCC parameters $\alpha_c$and $\beta_c$are significant in all three models. This result supports the time-varying correlation.

In the lower panel of Table 3, we report the model diagnostic statistics. The results are consistent with the presence of time-varying correlation and asymmetric dependence. Following Chen and Fan [2006], we perform the pseudo-likelihood ratio (PLR) test to show that the skewed t copula model outperforms the student t copula. The null hypothesis is that the asymmetry parameters (λ) in the skewed t copula are all zero. The pseudo-likelihood ratio (PLR) test rejects the null hypothesis. Thus, there is robust evidence of asymetry.

Figure 3 shows the dynamic correlation implied by the skewed t dynamic copula during the period from January 1 1989, to March 20 2020. The correlations of pairs HML&VAL and HML&MOM move around the value of Q (in equation 2). We consider the most difficult period of the recent financial crisis. During 2008, all pairs of correlations fluctuate considerably. The financial crisis hugely impacted on the forex market, invalidating models.

To reinforce our empirical results pointing towards non-normality and checking their robustness, in Figure 4 we plot the empirical threshold correlation of residuals $z^*$from the

[Figure 4Threshold Correlations for Factor Residuals and Copula Models]

AR-NGARCH model along with the standard bivariate normal implied threshold correlation, student t copula and skewed t copula implied threshold correlations. It is evident that the empirical threshold correlations are far from a bivariate normal distribution. In what follows we rely on the skewed t copula to model the dependency structure across factors in the forex market.

4 Portfolio Optimization

The empirical evidence above suggests that forex factors have significant time-varying asymmetric dependence. What is the economic cost of a forex trader ignoring this dependence structure? In the next sections, we shall consider forming optimal currency portfolios. We shall assess the economic value of considering this type of dependence structure in a forex portfolio. As in Kroencke et al. [2014], we use a real-time strategy. We show that once we implement a forex optimized strategy and consider asymmetry and time-varying in the dependence structure, the benefit in terms of utility is significant. For portfolio analysis, we assume that at each time t, investors allocate their wealth, based on the weighting vector w_t, across the 4 currency factors to maximize their expected utility. We compare the return characteristics of alternative strategies by using different dependence structure models and a large real time out-of-sample analysis. Note that following Christoffersen and Langlois [2013], we use the average return of the previous two years as a proxy for the expected return of the factors. This helps us focusing on the impact of higher moments on the portfolio selection.

4.1 The Investor's Optimization Problem

We assume that investors follow a constant relative risk aversion (CRRA) utility function:

$$U(\gamma) = \begin{cases} (1-\gamma)^{-1}\left(P_0\left(1+w_t^\top r_{t+1}\right)^{1-\gamma}\right) & if\, \gamma \neq 1 \\ log\left(P_0\left(1+w_t^\top r_{t+1}\right)\right) & if\, \gamma = 1 \end{cases}$$

Where P_0 is the initial wealth which we set at \$1 here, r_t is the vector 4 currency factor returns at time t, w_t is the weighting vector, γ denotes the degree of relative risk aversion

(RRA). We consider 3 levels of RRA: $\gamma = 3, 7, 10$. The weighting vector for each time t is obtained by maximizing the expected utility function which gives different assumptions for the factors' joint distribution.

$$
\begin{aligned}
w_t^* &\equiv \underset{w \in W}{arg\,max}\, E_{\hat{f}_{t+1}} \left(U \left(1 + w_t^\top r_{t+1} \right) \right) \\
&= \underset{w \in W}{arg\,max} \int \frac{\left(1 + w_t^\top r_{t+1}\right)^{1-\gamma}}{1-\gamma} f_{t+1}\left(r_{t+1}\right) dr_{t+1}
\end{aligned} \tag{3}
$$

Where $f_{t+1}(r_{t+1})$ denotes the joint distribution of the four factors. We assume that investors face investment constraints in that the risk exposure to any single factor and the four factors in total is less than \$1. Thus the weighting matrix $w = \{(w_1, w_2, w_3, w_4) \in [-1, 1]^4 :| w_1 | + | w_2 | + | w_3 | + |$ Due to the complexity of the joint distribution $f_{t+1}(r_{t+1})$, solution for w_t is generally not given analytically. We solved 3 by simulating 10,000 Monte Carlo replications for the four factors using a multivariate distribution $f_{t+1}(r_{t+1})$.

4.2 Forex Portfolio

Our weekly investment strategy is implemented in two stages: the first stage consists of modelling joint distribution for the expected return $f_{t+1}(r_{t+1})$; the second stage involves the estimation of the factor weighting vector by maximizing the investors' utility function 3. To begin with, we estimate the skewed t AR-NGARCH model for the four factors using the previous data sample following Cerrato et al. [2020]. Thereafter, we estimate the dependence structure between four residuals from the AR-NGARCH by using copula models.[10] Each time t, the expected factor return for factor j is generated by equation 4:

$$
r_{j,t+1} = \phi_{0,j} + \phi_{1,j} r_{j,t} + \sigma_{j,t+1} \epsilon_{j,t+1} \tag{4}
$$

Where $\phi_{o,j}$ and $\phi_{1,j}$ are the AR coefficients; $\sigma_{j,t+1}$ is the 1-step-ahead forecasted conditional volatility in the NGARCH model; $\epsilon_{j,t+1}$ is simulated from the joint distribution function which is characterized by the copula model. Note that the parameter estimates in the AR-NGARCH and copula models are updated once a year using the whole of the data sample. For

[10] We also used a multivariate standard normal distribution as a benchmark for comparison with copula models.

dynamic copula models, where DCC is used to model the time-varying correlation coefficient, the factor correlation is updated weekly. We start our investment from April 1, 1994, giving us an investment period of over 25 years.

In the second stage, we use the simulated 10,000 draws from $f_{t+1}(r_{t+1})$ to value the integral in 3. Thus maximizing 3 is equivalent to maximizing 5

$$w_t^* \equiv \underset{w \in W}{arg\,max} n^{-1} \sum_{i=1}^{n} U^* \left(R_{t+1,i}^* (w)\right) \tag{5}$$

where

$$R_{t+1,i}(w) = 1 + w_t^\top r_{t+1}$$

$$\varepsilon = 2.2204 \times 10^{-16}$$

and

$$R_{t+1,i}^*(w) = \begin{cases} R_{t+1,i}(w) & if\ R_{t+1,i}(w) > \varepsilon \\ 2\varepsilon \left(1 - \frac{1}{1+e^{R_{t,i}(w)-\varepsilon}}\right) & if\ R_{t+1,i}(w) \leq \varepsilon \end{cases}$$

$$\bar{U} = n^{-1} \sum_{i=1}^{n} U\left(R_{t,i}^*\left(w_{t-1}^*\right)\right)$$

$$U^*\left(R_{t+1,i}(w)\right) = \frac{100}{\mid \bar{U} \mid} U\left(R_{t+1,i}(w)\right)$$

The cut-off 2.2204×10^{-16} was chosen as the machine epsilon. We use the function U^* instead of U directly, since the numerical maximization routine does not work well with extremely small or large values. The $\bar{U}$does not affect the ranking of alternatives, and the 100 value is the reverting mean of the $\bar{U}$.

By maximizing equation 5, we obtain the optimal weighting vector w_t for time t. Each time t, investors liquidate the previous position and rebalance their portfolios according to w_t.

[The real-time investment results about here]

4.3 Performance of Different Strategies

The empirical results based on a large battery of dependence structure models are reported in Table 4. We consider three levels for the CRRA, namely $\gamma = 3$ in Panel A, $\gamma = 7$ in Panel B, and $\gamma = 10$ in Panel C. We follow Christoffersen and Langlois [2013], Patton [2004]. As the value of γ increases the risk-averse level also increases and the turnover decreases. The portfolio mean, volatility, skewness and kurtosis of returns for the 5 different models are given in Table 4. We start with the full dataset (i.e. developed and developing countries). Since we apply the average return, we apply the certainty equivalent to measure the performance of the portfolios.

We compute the certainty equivalent (CE) of the average realized utility for each strategy as follows

$$CE = U^{-1}\left(\frac{1}{T}\sum_{t=1}^{T}\frac{(1+r_{p,t})^{1-\gamma}}{1-\gamma}\right) = \left(\frac{1}{T}\sum_{t=1}^{T}(1+r_{p,t})^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

where U^{-1} is the inverse of the utility function and where

$$r_{p,t} = w_{t-1}^{\top} r_t$$

are the out-of-sample portfolio returns.

To find out whether richer models lead to better performance by generating a better trading signal, we also report the average turnover

$$Average\,turnover\,(\%) = \frac{100}{4T}\sum_{t=1}^{T}\sum_{i=1}^{4}|\ w_{i,t} - w_{i,t-1}\ |$$

The estimates are all around 12%-21%, depending on risk aversion. These values are similar within each of the panels. This indicates that the improvement in realized utility across the models is not driven by the difference in trading turnover.

[Table 5 Out-of-sample investment with transaction about here]

We use the multivariate standard normal model as our benchmark. Risk factors are assumed to be orthogonal with each other. Hence, we show the portfolios with the orthogonal assumption. There is clear empirical evidence that asymmetry is economically relevant (i.e. the skewed t copula out-performs the other models). Thus, by considering asymmetry one can add value to a forex portfolio.

To assess whether the difference between the benchmark portfolio and skew t-copula portfolio is economically significant, we apply bootstrap methods under the null hypothesis that the difference is significantly different from zero. In this way, we can infer if the actual difference shown in Table 4 is economically relevant.

4.4 Transaction Costs

Transaction costs can significantly reduce the performance of a trading strategy. There is empirical evidence [Menkhoff et al., 2012b], that the performance of a momentum strategy is highly reduced after considering transaction costs. In Table 5, we consider transaction costs to check the robustness of the results presented in the previous table. To compute the cost, we follow Barroso and Santa-Clara [2015]:

$$c_{i,t} = \frac{F^{ask}_{i,t,t+1} - F^{bid}_{i,t,t+1}}{F^{ask}_{i,t,t+1} + F^{bid}_{i,t,t+1}}$$

Where $c_{i,t}$ is the transaction cost of currency i at time t. $F^{ask}_{i,t,t+1}$ and $F^{bid}_{i,t,t+1}$ denote the bid and ask price of the forward exchange rate of currency i at time t. To convert currency transaction costs into factor transaction costs, we use the same method and parameters to calculate the factor transaction cost by simply changing currency excess return to the currency transaction cost.

We consider transaction costs for combined strategies and not a “stand alone” strategy as it may will be that when we consider transaction costs for a momentum strategy, the cost offsets the return for that strategy, but when it is combined with other strategies (for example carry trade) the higher profit of this combined strategy offsets the transaction costs. Clearly transaction costs are important. However, overall the main results remain unchanged. Moreover, the skewed t copula model shows stronger advantages.

[Table 6 Out-of-sample investment in developed currencies about here]

[Table 7 Out-of-sample investment in developing currencies about here]

4.5 Performance in Developed and Developing Countries

In this section, we split the data into developing and developed countries. Information on the dataset can be found in the footnote 1. We do this for several reasons: first, we aim to check whether our results are driven by country-specific factors affecting exchange rates. Second, the benefits of diversifying forex portfolios between developing and developed countries' exchange rates are well known. Table 6 shows that for the developed countries the p-values reject the null hypothesis only at the 10% significance level. Thus, the rejection is weaker than in the previous tables. The annualized mean return is generally higher for the t-skew copula model while annualized volatility and skewness stay unchanged across the models. The large negative skew may signal the presence of crash risk. As before, if we consider an investor with a relative risk aversion of 3, they would now gain 0.011% , this is 1.15bp per month if using the skew t-copula instead of our benchmark model.

The results for developing countries also point towards an economic gain when using a skew t-copula as opposed to our benchmark one, but they are weaker than those presented for all countries: the benefit for our investor of using a skew t-copula model, in this case, is only 1.94bp per month. There is an economic benefit in diversifying an forex portfolio between developed and developing markets. The annualized mean return for developing countries is higher than that for developed countries, but annualized volatility is also higher. Overall, the CE measure for developing countries is the highest.

5 Risk Management

In this section, we consider the implications for risk management when we fail to correctly specify the relevant risk measeres. We compute two risk measures for each factor: the Value at Risk (VaR), which is the tail quantile of the conditional distribution of the portfolio returns; and the expected short (ES) fall, which is the conditional expectation of exceeding the VaR. If there is a loss function which could calculate the risk measure by minimizing the expected loss of the function, the risk measure is elicited [Patton et al., 2019]. Note that the risk measure from loss function is the exact risk of the variable. For instance, the mean value could be obtained using the quadratic loss function, and the VaR value could be obtained using the piecewise linear function.

Fissler [2017] shows the joint elicitation of the VaR and ES in the loss function. We follow the loss function from Fissler [2017] to forecast the VaR and ES and evaluate the performance of the forecasting models. In this section, we follow Patton et al. [2019] univariate distribution model and extend it to a multivariate setting by using copula models.

In risk forecasting, we apply the in-sample data from 1 January 1989 to 10 March, 2005, and out-of-sample data from 11 March, 2005 to 20 March 2020. Following Patton et al. [2019], we estimate the model only once on 10 March, 2005. The future risk measures are forecast from the updated data and the consistent model. We will apply the goodness-of-fit test and Diebold-Mariano tests to evaluate the performance of the forecasting models.

5.1 Copula VaR and ES Forecasting

We apply the distribution of factor returns, the mean and variance, to forecast the VaR and ES. We first use GARCH dynamics, following Patton et al. [2019], for the conditional mean and variance to get the volatility item σ_t. The copula forecasting model is:

$$Y_t = \mu_t + \sigma_t \eta_t$$

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \gamma \eta_{t-1}^2$$

Y_t denotes the factor portfolio's return, where μ_t is specified to the ARMA model and σ_t^2 is specified to the GARCH model. η_t denotes the residual of the factor returns. Given F_η, the forecasting of VaR and ES can be estimated as:

$$v_t = \mu_t + a\sigma_t, \ where\, a = F_\eta^{-1}(\alpha)$$

$$e_t = \mu_t + b\sigma_t, \ where\, b = \mathbb{E}\left[\eta_t \mid \eta_t \leq a\right]$$

In the univariate model, Patton et al. [2019] assume the residuals η_t follows the univariate distribution. We, however, assume the η_t follows the joint distribution as indicated here. We consider three choices for F_η to describe the distributions of η_t:

$$\eta_t \sim iid\, Normal\, linear$$

$$\eta_t \sim iid\, orthoganal$$

$$\eta_t \sim iid\, Normal\, copula$$

$$\eta_t \sim iid\, Student\, t\, copula$$

$$\eta_t \sim iid\, Skewed\, t\, copula$$

To estimate the parameters (a, b), we use the Monte Carlo simulation. We use simulation to define the quantile function and employ 1,000,000. Thereafter, we sort the replications to obtain the quantile value a.

We apply the NGARCH model in Section 3.2 to get the residual of the factor portfolios. The details of the univariate model are shown in the Appendix A. To keep consistency with the previous model, we also use the NGARCH model to consider leverage effects in forecasting

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\sigma_{t-1}^2(\eta_{t-1} - \theta)^2$$

In the next subsection, we shall discuss the goodness-of-fit and Diebold-Mariano tests as in Patton et al. [2019].

5.2 Ranking from different test

Table 8 shows the fit of the future risk measures for different forecasting models. The good-of-fit test using the FZ-loss function follows Fissler [2017]. That is, minimizing the expected loss using any of these loss functions returns the true VaR and ES. The loss function is shown below:

$$L_{FZ}(Y,v,e,G_1,G_2) = \underset{(v,e)}{argmin}\left(1\left\{Y\leq v\right\}-\alpha\right)\left(G_1\left(v\right)-G_1\left(Y\right)+\frac{1}{\alpha}G_2\left(e\right)v\right)$$
$$-G_2\left(e\right)\left(\frac{1}{\alpha}1\left\{Y\leq v\right\}Y-e\right)-\mathcal{G}_2\left(e\right)$$

where G_1 denotes the weakly increasing and G_2 denotes the strictly increasing and strictly positive. To simplify the loss function, Patton et al. [2019] assume that the loss differences from the loss function are homogeneous of degree zero. Thus, the G_1 and G_2 will be $G_1(x) = 0\, and\, G_2(x) = -1/x$.[11] G_2 is the differential coefficient function of $\mathcal{G}_2$, $\mathcal{G}_2' = G_2$. Parameters v, e denote the VaR and ES.

Following Patton et al. [2019], the loss function can be rewritten in the following form:

$$L_{FZ0}(Y,v,e;\alpha) = \frac{1}{\alpha e}1\left\{Y\leq v\right\}\left(v-Y\right)+\frac{v}{e}+log\left(-e\right)-1$$

We carry out the goodness-of-fit test and Diebold-Mariano tests as in Patton et al. [2019]. The two tests are briefly discussed below. The idea of testing is that the expected value at time of the loss function partial derivative for VaR and ES should equal to zero:

$$\mathbb{E}_{t-1}\begin{bmatrix}\partial L_{FZ0}\left(Y_t,v_t,e_t;\alpha\right)/\partial v_t\\ \partial L_{FZ0}\left(Y_t,v_t,e_t;\alpha\right)/\partial e_t\end{bmatrix}=0 \tag{6}$$

When the partial derivatives equal to zero, it indicates a good results for the goodness-of-fit test. To simplify the calculation, let $\lambda_{v,t}^s = \partial L_{FZ0}\left(Y_t,v_t,e_t;\alpha\right)/\partial v_t$ and $\lambda_{e,t}^s = \partial L_{FZ0}\left(Y_t,v_t,e_t;\alpha\right)/\partial e_t$.

$$\lambda_{v,t}^s \equiv \tfrac{\lambda_{v,t}}{v_t} = 1\left\{Y_t\leq v_t\right\}-\alpha$$
$$\lambda_{e,t}^s \equiv \tfrac{\lambda_{e,t}}{e_t} = \tfrac{1}{\alpha}1\left\{Y_t\leq v_t\right\}\tfrac{Y_t}{e_t}-1$$

Consequently, the standardized and generalized residuals also have the expected value zero, $\mathbb{E}_{t-1}\left[\lambda_{v,t}^s\right] = \mathbb{E}_{t-1}\left[\lambda_{e,t}^s\right] = 0$. Patton et al. [2019] uses the dynamic quantile (DQ) approach employing a simple regression of the generalized residuals at each term t. To test $E\left[\lambda_{v,t}^s\right] = \mathbb{E}_{t-1}\left[\lambda_{e,t}^s\right] = 0$, we use the following regressions:

[11] The detailed proof for the choosing of G_1 and G_2 can be found in Patton et al. [2019].

[Table 8: Rank of Diebold-Mariano tests and goodness-of-fit test of copula forecasting models]

$$\lambda^s_{v,t} = a_0 + a_1\lambda^s_{v,t-1} + a_2 v_t + u_{v,t}$$
$$\lambda^s_{e,t} = b_0 + b_1\lambda^s_{e,t-1} + b_2 e_t + u_{e,t}$$

If the parameters in the regression are all zero, the results show that the forecasting risk measures pass the test. Following Patton et al. [2019], we set the hypothesis that all parameters in the regressions are not zero. In table8, we show the p-value of the goodness-of-fit. The results which do not pass the test are in bold.

Diebold-Mariano tests compare the average loss of each model statistically to show which achieves the best performance. The tables in the Appendix B, Appendix C and Appendix D show the Diebold-Mariano tests results. A positive value indicates that the column model outperforms the row model on the average loss using the loss function from Fissler [2017][12]. In the Appendix, we show that the t-statistics from Diebold-Mariano tests comparing the average losses over an out-of-sample period from 11 March, 2005, to 20 March, 2020, for 10 different forecasting models. We present a summary for the Diebold-Mariano tests in the table 8.

As we are focusing on multivariate risk forecasting, we apply the average rank of the four factors as the ratio to compare the performance of the models. The GARCH skewed t copula model shows the best performance among the 10 models in the cross section (whole data set) and the developed countries dataset, while the NGARCH skewed t copula model ranks first among the models in the developing countries factors. Furthermore, the benchmark models (orthogonal and normal linear distribution) always underperform the DCC-copula model in forecasting risk.

The results confirm our findings in the portfolio management section. These models can help to accommodate extreme changes. The factors MOM and VAL carry the highest risk. The models accounting for asymmetry generally carry the lowest average loss. Our results show that asymmetry between forex factors is an important element and can help improving the risk management of forex portfolios.

[12] We follow Patton et al. [2019] to apply the average loss as the rank ratio

6 Conclusion

We conduct a detailed study on the dependence structure among the most widely investigated currency factors in the literature that are also very relevant to the hedge fund industry when designing forex trading strategies. We show that the dependence structure between forex factors is more complex than has been considered in the literature so far and asymmetry and time dependence are very relevant. To evaluate the economic cost to a hedge fund of neglecting these modelling features, we consider two examples: forex portfolio management and forex portfolio risk management and show that adding asymmetry and time-varying dependence among the factors improves portfolio performance and risk management. Our results are very relevant for the academic literature in this area as we shed some new light on the dependence structure between some popular forex factors, and they are also relevant to hedge funds when designing forex trading strategies.

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Figure 1 – Quantile-Quantile Plots for 4 factors

For each observation, we scatter plot the empirical quantile on the vertical axis against the corresponding quantile from the standard normal distribution on the horizontal axis. If returns are normally distributed, then the data points will fall randomly around the 45° line ,which is marked by dashes.

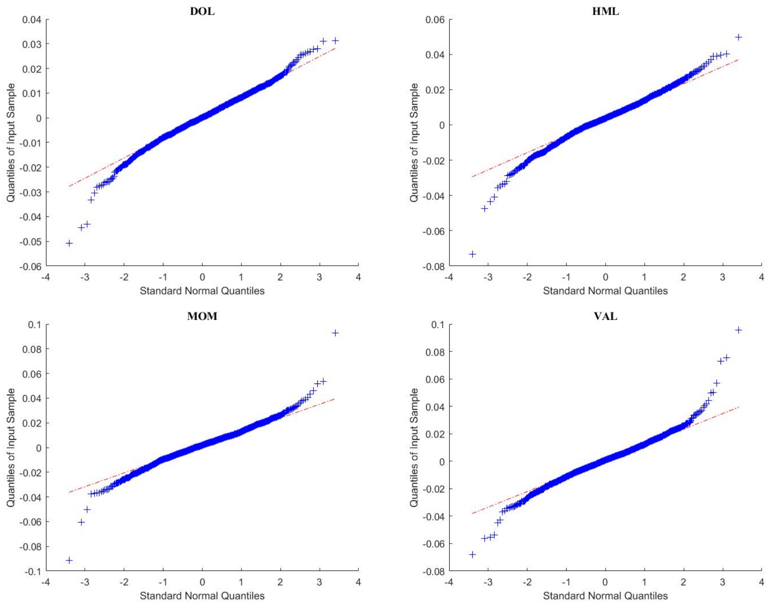

Figure 2 – Threshold correlation for 4 factors

This figure presents threshold correlations between the 4 factors . Our sample consists of weekly returns from January 1, 1989, to March 20, 2020. The continuous line represents the correlations when both variables are below (above), a threshold when this threshold is below (above) the median. The dashed line represents the threshold function for a bivariate normal distribution using the linear correlation coefficient from the data.

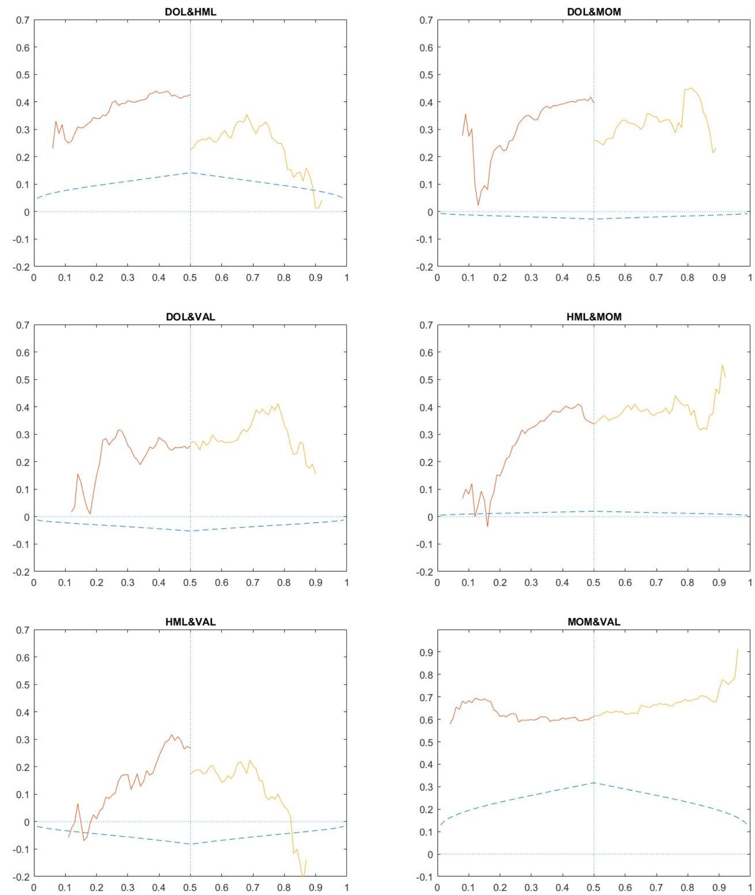

Figure 3 – Skewed t copula dynamic correlations with composite method

We report dynamic conditional copula correlation for each pair of factors from January 1, 1989, to March 20, 2020. The correlations are obtained by estimating the dynamic skewed t copula model on the factor return residuals from the AR-GARCH model. This sample is used in estimation of the models.

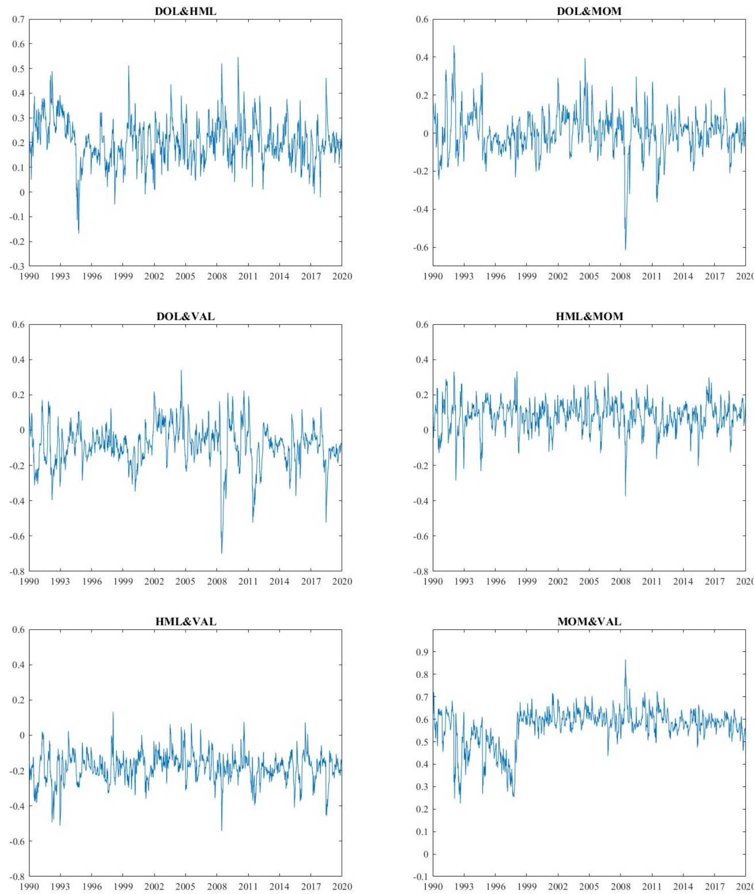

Figure 4 – Threshold Correlations for Factor Residuals and Copula Models

We present threshold correlations computed on AR-GARCH residuals from January 1, 1989, to March 20, 2020. The thick continuous line represents the empirical correlation. The threshold correlation functions are computed for thresholds for which there are at least 24 data points available. We compared the empirical correlations to those implied by the normal copula and the constant t and skewed t copulas.

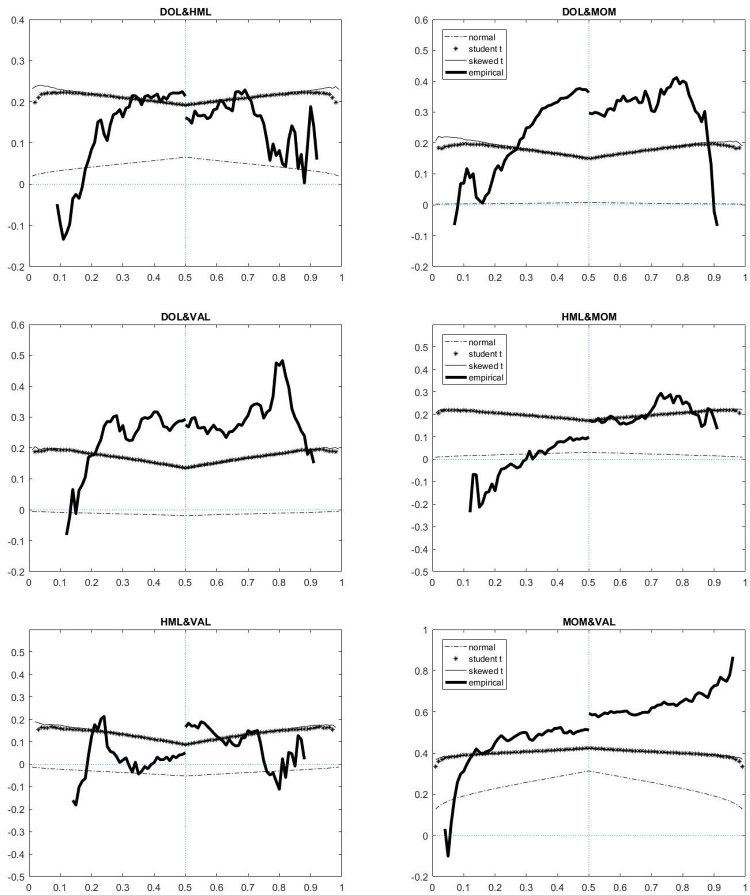

Table 1 – Description Statistics of Weekly Factor Return

We report the annualized mean, annualized volatility, skewness, kurtosis and autocorrelation and cross-correlation for logged weekly return of four factors. The period of the sample is from January 1, 1989, to March 20, 2020. The significant correlation is marked by * and ** denoting the 5% and 1% levels.

Sample Moments	DOL	HML	MOM	VAL
Mean	0.00	0.18	0.08	0.03
Volatility	0.06	0.08	0.09	0.10
skewness	-0.35	-0.40	-0.15	0.26
Kurtosis	4.73	5.16	6.93	6.86
Autocorrelation				
First-order	0.04	-0.01	-0.01	0.09**
Second-order	0.04	0.09**	0.06*	0.18**
Third-order	0.02	0.08**	0.09**	0.14**
Cross Correlations				
DOL	1.00	0.31**	-0.08*	-0.16**
HML	0.31**	1.00	0.05*	-0.27**
MOM	-0.08*	0.05*	1.00	0.56**
VAL	-0.16**	-0.27**	0.56**	1.00

Table 2 – Different group and period of four factors' correlations

We present the different group and period correlations to understand the reason for the positive correlation between HML and MOM or negative correlation between HML and VAL. The first section presents the correlation of the group of developed country factors, while the second section show the correlations from developing country factors. The datasets of the developed and developing countries are shown in the footnote 1. The last section is the cross-section data without the 2008 financial crisis, as we exclude the data from Sep, 2007 to Sep, 2009.

	developed countries				developing countries				without financial crisis data			
	DOL	HML	MOM	VAL	DOL	HML	MOM	VAL	DOL	HML	MOM	VAL
DOL	1.00	0.28	-0.14	-0.06	1.00	0.40	-0.05	-0.00	1.00	0.29	-0.06	-0.18
HML	0.28	1.00	-0.16	0.01	0.40	1.00	0.22	0.31	0.29	1.00	0.07	-0.33
MOM	-0.14	-0.16	1.00	0.14	-0.05	0.22	1.00	0.32	-0.06	0.07	1.00	0.43
VAL	-0.06	0.01	0.14	1.00	-0.00	0.31	0.32	1.00	-0.18	-0.33	0.43	1.00

Table 3 – Estimation results for copula models with composite method

This table presents parameter estimates for the dependence models of the residuals from the NAGARCH model for the period January 1, 1989, to March 20, 2020. All models are estimated by maximum likelihood. Standard errors (in parentheses) are computed using the methodology of Engle et al. [2009]. The last line presents the pseudo-likelihood ratio test statistics. We followed Chen and Fan [2006] for the null hypothesis that the asymmetry parameters in skewed t copula are all equal to 0. The * and ** denote the significant levels of 5% and 1%. The value and the standard errors of the λ are multiplied 100.

	4 factors					
	constant			dynamic		
	normal	t	skewed t	normal	t	skewed t
υ		9.16	7.25		6.07	6.49
		(2.26)	(0.13)		(0.94)	(0.28)
λ_{DOL}			-0.09			-0.25
			(0.02)			(0.13)
λ_{HML}			0.15			0.31
			(0.05)			(0.07)
λ_{MOM}			-0.13			-0.22
			(1.62)			(0.51)
λ_{VAL}			-0.95			-0.02
			(0.26)			(1.06)
β_c				0.81	0.80	0.81
				(0.03)	(0.01)	(0.02)
α_c				0.02	0.04	0.03
				(0.01)	(0.00)	(0.00)
ρ(DOL,HML)	0.16	0.15	0.15	0.21	0.21	0.21
ρ(DOL,MOM)	0.02	0.04	0.04	0.02	-0.00	-0.00
ρ(DOL,VAL)	-0.05	-0.01	-0.00	-0.05	-0.10	-0.09
ρ(HML,MOM)	0.08	0.09	0.09	0.11	0.08	0.08
ρ(HML,VAL)	-0.16	-0.16	-0.16	-0.18	-0.19	-0.19
ρ(MOM,VAL)	0.55	0.56	0.56	0.55	0.57	0.57
Model Properties						
Correlation persistence	0.00	0.00	0.00	0.84	0.83	0.84
Log-likelihood	334.99	544.75	551.96	521.36	602.80	610.38
No. of parameters	6.00	7.00	11.00	8.00	9.00	13.00
Pseudo-likelihood			15.28**			12.47**

Table 4 – Out-of-sample investment results

The period of the out-of-sample is from April 1, 1994, to March 20, 2020. For each level of relative risk aversion, the performance of the three copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. We report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

	Dynamic correlation models				
	Linear		Copula		
	Orthogonal	Normal	Normal	Student t	Skewed t
Panel A. γ=3					
Return(%)	18.58	18.60	18.59	18.60	18.89
Volatility(%)	12.90	12.89	12.90	12.89	12.74
skewness	-0.14	-0.15	-0.15	-0.15	-0.03
Kurtosis	2.90	2.92	2.93	2.93	2.58
Average turnover(%)	1.37	1.81	1.88	1.93	1.71
CE(basis point)	35.47	35.70	35.67	35.70	36.25
Diff in CE(%)	–	–	–	–	0.28
Bootstrap p-value	–	–	–	–	0.01
Panel B. γ=7					
Return(%)	18.23	18.45	18.43	18.53	18.69
Volatility(%)	12.92	12.91	12.92	12.91	12.72
skewness	-0.07	-0.13	-0.13	-0.15	0.00
Kurtosis	2.90	2.95	2.95	2.95	2.66
Average turnover(%)	1.40	2.19	2.24	2.22	2.13
CE(basis point)	34.79	35.29	35.26	35.45	35.76
Diff in CE(%)	–	–	–	–	0.24
Bootstrap p-value	–	–	–	–	0.03
Panel C. γ=10					
Return(%)	18.08	18.31	18.31	18.46	18.64
Volatility(%)	12.98	12.96	12.96	12.97	12.94
skewness	-0.06	-0.13	-0.13	-0.16	-0.14
Kurtosis	2.85	2.92	2.92	2.91	2.85
Average turnover(%)	1.41	2.20	2.19	2.22	2.35
CE(basis point)	34.50	34.94	34.94	35.23	35.58
Diff in CE(%)	–	–	–	–	0.33
Bootstrap p-value	–	–	–	–	0.01

Table 5 – Out-of-sample investment with transaction cost

This table has same structure as table 4. However, we show the out-of-sample investment with the transaction cost.

	Dynamic correlation models				
	Linear		Copula		
	Orthogonal	Normal	Normal	Student t	Skewed t
Panel A. γ=3					
Return(%)	17.75	17.74	17.70	17.69	18.07
Volatility(%)	13.21	13.19	13.20	13.21	12.98
skewness	-0.22	-0.20	-0.20	-0.21	-0.06
Kurtosis	2.98	2.97	2.96	2.99	2.61
Average turnover(%)	1.72	1.81	1.88	1.93	1.71
CE(basis point)	33.93	34.02	33.95	33.94	34.67
Diff in CE(%)	–	–	–	–	0.33
Bootstrap p-value	–	–	–	–	0.01
Panel B. γ=7					
Return(%)	17.38	17.35	17.31	17.40	17.60
Volatility(%)	13.20	13.26	13.28	13.24	13.01
skewness	-0.13	-0.17	-0.17	-0.17	-0.01
Kurtosis	2.94	2.95	2.96	2.93	2.73
Average turnover(%)	1.75	2.19	2.24	2.22	2.13
CE(basis point)	33.23	33.16	33.08	33.25	33.64
Diff in CE(%)	–	–	–	–	0.25
Bootstrap p-value	–	–	–	–	0.01
Panel C. γ=10					
Return(%)	17.23	17.19	17.19	17.32	17.44
Volatility(%)	13.24	13.30	13.30	13.29	13.28
skewness	-0.11	-0.17	-0.16	-0.17	-0.16
Kurtosis	2.90	2.92	2.91	2.88	2.83
Average turnover(%)	1.76	2.20	2.19	2.22	2.35
CE(basis point)	32.93	32.76	32.75	33.01	33.24
Diff in CE(%)	–	–	–	–	0.25
Bootstrap p-value	–	–	–	–	0.02

Table 6 – Out-of-sample investment in developed countries currencies

This table has the same structure as Table 4. This table shows the out-of-sample investment with the developed countries' factor portfolios.

	Dynamic correlation models				
	Linear		Copula		
	Orthogonal	Normal	Normal	Student t	Skewed t
Panel A. γ=3					
Return(%)	16.54	16.55	16.54	16.50	16.68
Volatility(%)	13.51	13.51	13.51	13.52	13.50
skewness	-0.49	-0.49	-0.49	-0.49	-0.50
Kurtosis	2.90	2.90	2.90	2.89	2.90
Average turnover(%)	2.81	2.80	2.81	2.66	2.55
CE(basis point)	31.72	31.72	31.72	31.64	31.99
Diff in CE(%)	–	–	–	–	0.14
Bootstrap p-value	–	–	–	–	0.08
Panel B. γ=7					
Return(%)	16.26	16.254	16.26	16.18	16.27
Volatility(%)	13.50	13.50	13.50	13.55	13.50
skewness	-0.45	-0.44	-0.45	-0.45	-0.45
Kurtosis	2.88	2.88	2.88	2.84	2.87
Average turnover(%)	2.79	2.82	2.79	2.81	2.40
CE(basis point)	31.05	31.04	31.05	30.89	31.06
Diff in CE(%)	–	–	–	–	0.01
Bootstrap p-value	–	–	–	–	0.08
Panel C. γ=10					
Return(%)	16.15	16.14	16.15	16.08	16.16
Volatility(%)	13.54	13.54	13.54	13.57	13.57
skewness	-0.43	-0.43	-0.43	-0.42	-0.42
Kurtosis	2.83	2.83	2.83	2.80	2.80
Average turnover(%)	2.71	2.74	2.71	2.73	2.47
CE(basis point)	30.73	30.72	30.73	30.60	30.75
Diff in CE(%)	–	–	–	–	0.02
Bootstrap p-value	–	–	–	–	0.03

Table 7 – Out-of-sample investment in developing countries currencies

This table has the same structure as Table 4. This table shows the out-of-sample investment with the developing countries' factor portfolios.

	Dynamic correlation models				
	Linear		Copula		
	Orthogonal	Normal	Normal	Student t	Skewed t
Panel A. γ=3					
Return(%)	23.78	23.87	23.87	23.86	23.92
Volatility(%)	15.08	15.10	15.09	15.10	15.09
skewness	-0.02	-0.05	-0.04	-0.05	-0.05
Kurtosis	2.71	2.74	2.74	2.74	2.73
Average turnover(%)	1.61	1.95	2.00	2.03	1.88
CE(basis point)	45.59	45.40	45.76	45.74	45.85
Diff in CE(%)	–	–	–	–	0.23
Bootstrap p-value	–	–	–	–	0.01
Panel B. γ=7					
Return(%)	23.71	23.78	23.78	23.80	23.84
Volatility(%)	15.23	15.23	15.23	15.22	15.21
skewness	-0.09	-0.11	-0.11	-0.12	-0.11
Kurtosis	2.71	2.77	2.77	2.78	2.79
Average turnover(%)	1.70	2.39	2.38	2.46	2.38
CE(basis point)	45.24	45.37	45.37	45.40	45.49
Diff in CE(%)	–	–	–	–	0.03
Bootstrap p-value	–	–	–	–	0.06
Panel C. γ=10					
Return(%)	23.83	23.91	23.91	23.92	24.00
Volatility(%)	15.22	15.18	15.18	15.18	15.17
skewness	-0.12	-0.15	-0.15	-0.15	-0.16
Kurtosis	2.83	2.94	2.94	2.96	2.97
Average turnover(%)	1.69	2.60	2.61	2.68	2.64
CE(basis point)	45.31	45.47	45.47	45.48	45.64
Diff in CE(%)	–	–	–	–	0.09
Bootstrap p-value	–	–	–	–	0.07

Table 8 – Rank of Diebold-Mariano tests and goodness-of-fit test of copula forecasting models

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler [2017], for four factors return series, over the out-of-sample period from January 7 1994 to January 5 2018, for ten different forecasting models. The first five rows correspond to dynamic copula forecasts, the last five rows give the forecasting results from NAGARCH dynamic copula models.The left panel shows the rank of Diebold-Mariano tests. The detail of Diebold-Mariano tests is shown in the Appendix. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level are in bold. The last column shows the average rank of the four factors. The panel.A shows the test results of the whole data set. The panel.B presents the results of the developed countries' dataset, while panel.C gives the results of the developing countries dataset.

	Rank from test				Goodness-of-fit-VaR				Goodness-of-fit-ES				Average rank of the models
Panel.A cross section	DOL	HML	MOM	VAL	DOL	HML	MOM	VAL	DOL	HML	MOM	VAL	Amongst 4 factors
GCH-n-dcc	1	8	10	3	**0.91**	0.00	0.00	0.00	**0.63**	0.00	0.00	0.00	5.5
GCH-t-dcc	4	2	1	5	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	3
GCH-skt-dcc	5	1	2	4	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	3
GCH-bm-dcc	2	10	9	1	**0.91**	0.00	0.00	0.00	**0.58**	0.00	0.00	0.00	5.5
GCH-orth-dcc	3	9	8	2	**0.91**	0.00	0.00	0.00	**0.58**	0.00	0.00	0.00	5.5
NGCH-n-dcc	9	6	7	9	0.09	0.00	0.00	0.01	0.08	0.01	0.00	0.00	7.75
NGCH-t-dcc	6	4	4	6	0.08	0.00	0.00	0.02	0.09	0.00	0.00	0.00	5
NGCH-skt-dcc	7	3	3	7	0.08	0.00	0.00	0.01	0.07	0.00	0.00	0.00	5
NGCH-bm-dcc	8	5	5	8	0.09	0.00	0.00	0.01	0.08	0.00	0.00	0.00	6.5
NGCH-orth-dcc	10	7	6	10	**0.12**	0.00	0.00	0.01	0.08	0.01	0.00	0.00	8.25
Panel.B developed countries													
GCH-n-dcc	3	5	4	2	**0.13**	0.00	0.00	**0.91**	**0.14**	0.00	0.00	**0.89**	3.5
GCH-t-dcc	4	2	2	5	0.06	0.00	0.00	**0.28**	0.04	0.00	0.00	**0.58**	3.25
GCH-skt-dcc	5	1	1	1	0.01	0.00	0.00	**0.55**	0.00	0.00	0.00	**0.80**	2
GCH-bm-dcc	1	3	5	4	**0.13**	0.00	0.00	**0.89**	**0.14**	0.00	0.00	**0.84**	3.25
GCH-orth-dcc	2	4	3	3	**0.13**	0.00	0.00	**0.91**	**0.14**	0.00	0.00	**0.89**	3
NGCH-n-dcc	8	6	8	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.5
NGCH-t-dcc	6	9	7	10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8
NGCH-skt-dcc	7	10	6	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8
NGCH-bm-dcc	9	8	9	7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.25
NGCH-orth-dcc	10	7	10	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.25
Panel.B developing countries													
GCH-n-dcc	4	10	7	6	**0.82**	0.00	0.00	0.08	**0.74**	0.00	0.00	0.01	6.75
GCH-t-dcc	3	7	4	4	**0.65**	0.02	0.00	**0.25**	**0.67**	0.02	0.00	0.05	4.5
GCH-skt-dcc	6	6	3	5	0.00	0.04	0.00	**0.14**	0.04	0.04	0.00	0.02	5
GCH-bm-dcc	5	8	6	7	**0.82**	0.00	0.00	0.04	**0.74**	0.00	0.00	0.00	6.5
GCH-orth-dcc	7	9	5	8	**0.82**	0.00	0.00	0.04	**0.74**	0.00	0.00	0.00	7.25
NGCH-n-dcc	10	1	9	10	0.00	**0.13**	0.00	0.00	**0.59**	0.08	**0.55**	**0.25**	7.5
NGCH-t-dcc	1	4	8	2	0.00	**0.30**	0.00	0.00	**0.66**	**0.59**	**0.76**	0.07	3.75
NGCH-skt-dcc	2	5	2	1	0.00	**0.30**	0.00	0.00	**0.78**	**0.57**	**0.65**	**0.71**	2.5
NGCH-bm-dcc	8	3	1	9	0.00	**0.13**	0.00	0.00	**0.80**	0.08	**0.73**	**0.26**	5.25
NGCH-orth-dcc	9	2	10	3	0.00	**0.13**	0.00	0.00	**0.79**	0.08	**0.55**	**0.36**	6

Appendix A Univariate model

The empirical results in Table 1 also show that autocorrelation could be an important issue for factors' returns. In Figure 5, the autocorrelation function is plotted by a dashed line for all the factors up to 100 lags, a 95% confidence boundary included. Financial time series are generally subject to heteroscedasticity and volatility clustering. We plot the autocorrelation function for the absolute value of the factors on the same graph. We find a strong and persistent serial correlation.

We model the dynamics of factors by using a univariate autoregressive-non-linear generalized autoregressive conditional heteroscedasiticity (AR-NGARCH) process. The conditional mean is estimated by an AR(1) process as follows:

$$r_{j,t} = \phi_{0,j} + \phi_{i,j} r_{j,t-1} + \sigma_{j,t}\epsilon_{j,t} \tag{7}$$

Where $r_{j,t}$is the factor value of factor j at time t. The conditional volatility is governed by an NGARCH [Engle and Ng, 1993]

$$\sigma^2_{j,t} = \omega_j + \beta_j \sigma^2_{j,t-1} + \alpha_j \sigma^2_{j,t-1}(\epsilon_{j,t-1} - \theta_j)^2 \tag{8}$$

The NGARCH model allows for the asymmetric influence of past return innovations $\epsilon_{j,t-1}$. Since financial time series generally show a “leverage effect”, an unexpected drop in return may have a bigger impact on conditional volatility than an unexpected increase (i.e. $\theta_j$is positive). Under this circumstance, the NGARCH model is expected to mitigate the skewness and excess kurtosis. We use the maximum likelihood method under the assumption of i.i.d. normal innovations of $\epsilon_{j,t}$.

Table 9 reports the coefficient estimates and diagnostic tests under the normal assumption for $\epsilon_{j,t}$. In the first panel, we report the estimated coefficients and standard errors of an AR(1)-NGARCH model ϕ_0,ϕ_1,α,β and θ. The parameters (ϕ_0) are all significant except for the DOL. Most parameters of the NGARCH model are also significant. The coefficient θ of the VAL and the MOM factors have large positive values which are statistically significant while the DOL factors have insignificant negative θ. The log-likelihoods are all significant and positive.

The divergence between model skewness/kurtosis points towards strong non-normality of ϵ_j.To better model the factor dynamics, we employ the skewed t distribution of Hansen [1994] for error term $\epsilon_{j,t}$, where the coefficients $\kappa_j$and $\upsilon_j$govern the skewness and the kurtosis. We use the maximum likelihood method under the assumption of skewed t distribution of $\epsilon_{j,t}$to estimate the AR(1)-NGARCH model. The results are reported in Table 10 which shows that the kurtosis parameters (υ) are all significant and the skewness factors (κ) of HML are not significant.[13]

Figure 6 graphs the autocorrelation function for the residual and its absolute value. After adjusting the skewness and excess kurtosis by assuming a normal distribution, the serial correlation in absolute value is highly reduced. Figure 7 is the QQ plot of the residuals from skewed t AR(1)-NGARCH. When comparing these results with Figure 1, we see that most of the skewedness and kurtosis have been modelled after using the AR(1)-NGARCH.

[13]By comparing the significance for the whole AR(1)-NGARCH model in Table 10, we find that the AR(1)-NGARCH model with the normal distribution fits the data well.

Figure 5 – Autocorrelation 4 factors and the absolute value of 4 factors

Autocorrelation of weekly returns (dashed line) and absolute returns (solid line) from January 1, 1989, to March 20, 2020. The horizontal dotted lines provide a 95 confidence interval around 0.

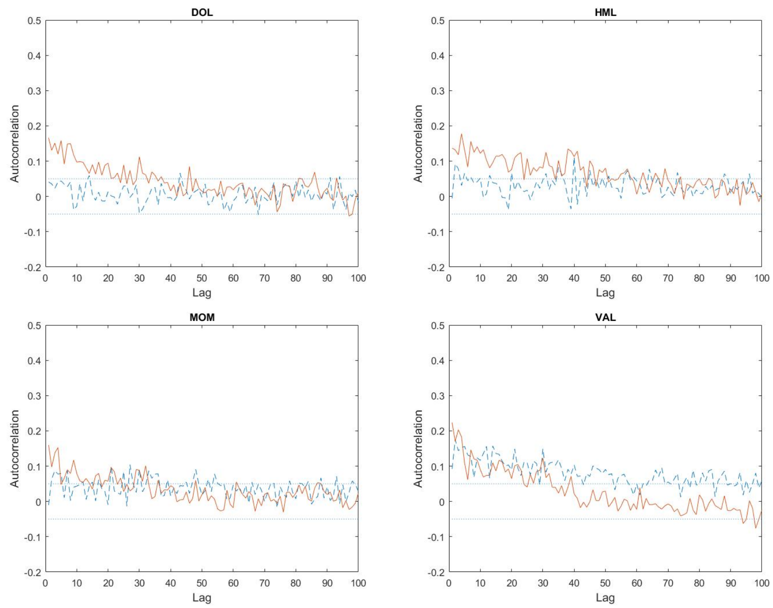

Figure 6 – Autocorrelation graph of residual series

Autocorrelation of AR-FARCH residuals (dashed line) and absolute residuals (solid line) from January 1, 1989, to March 20, 2020. The horizontal dotted lines provide a 95 confidence interval around 0.

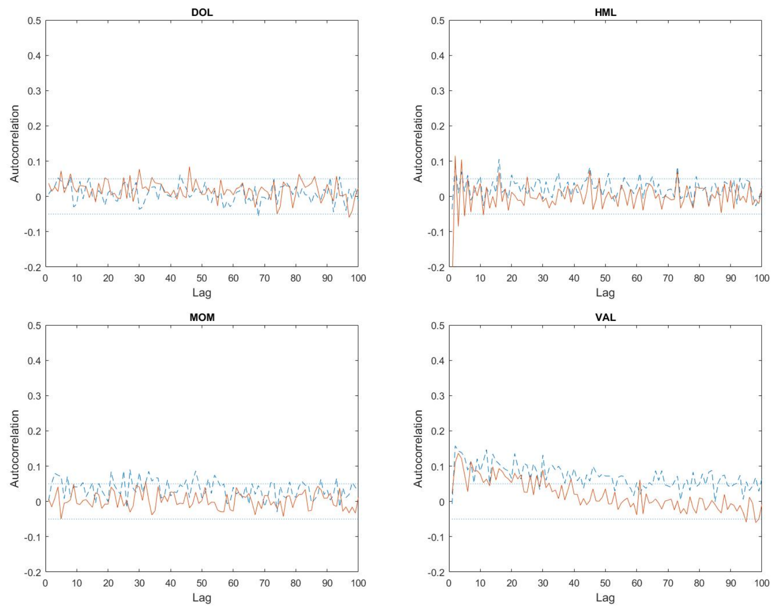

Figure 7 – QQ plot of residuals series

For each observation we scatter plot the empirical quantile on the vertical axis against the corresponding quantile from the skewed t distribution on the horizontal axis. If the AR-GARCH residuals adhere to the skewed t distribution, then the data points will fall on the 45º line, which is marked by dashes. The parameters for the skewed t distribution are from Table 3.

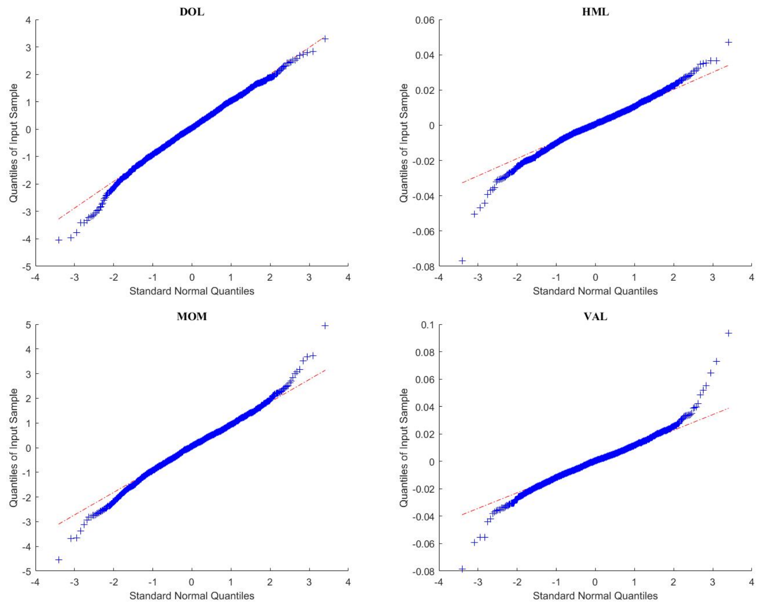

Table 9 – Estimation table of normal residuals

We report parameter estimates and model diagnostics for the AR-NGARCH model with normal shocks. Standard errors which are in parentheses are calculated from the outer product of the gradient at the optimum parameter values. The model estimated is $r_{j,t} = \phi_{0,j} + \phi_{i,j} r_{j,t-1} + \sigma_{j,t}\epsilon_{j,t}$, where $\sigma^2_{j,t} = \omega_j + \beta_j \sigma^2_{j,t-1} + \alpha_j \sigma^2_{j,t-1}(\epsilon_{j,t-1} - \theta_j)^2$. Here ω is fixed by variance targeting, and variance persistence denotes the sum of parameters of the model.We also provide the p-value for Ljung-Box (L-B) tests of the residuals and absolute residuals by 20 lags. The empirical skewness and excess kurtosis of the residuals are compared to the model implied levels from the normal model.

Parameter Estimates	DOL	HML	MOM	VAL
ϕ_0	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	0.04	0.02	0.01	0.11
	(0.04)	(0.03)	(0.03)	(0.03)
α	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
β	0.93	0.95	0.91	0.00
	(0.56)	(0.02)	(0.09)	(0.00)
θ	-0.37	0.08	0.49	0.12
	(0.22)	(0.12)	(0.11)	(0.02)
κ	/	/	/	/
υ	/	/	/	/
Diagnostics				
Log-likelihood	5131.10	4714.60	4572.50	4451.70
Variance persistence	0.93	0.95	0.91	0.00
L-B(20) p-value	0.14	0.00	0.00	0.00
Absolute L-B(20) p-value	0.00	0.00	0.00	0.00
Empirical skewness	-0.33	-0.39	-0.16	0.15
Model skewness	0.00	0.00	0.00	0.00
Empirical excess kurtosis	4.71	5.17	7.03	6.91
Model excess kurtosis	0.00	0.00	0.00	0.00

Table 10 – Estimation table of skewed t residuals

This table show the same structure as Table 9. We report parameter estimates and model diagnostics for the AR-NGARCH model with skewed t shocks.

Parameter Estimates	DOL	HML	MOM	VAL
ϕ_0	0.01	0.019	0.01	0.00
	(4.43)	(39.88)	(19.04)	(0.00)
ϕ_1	0.12	-0.99	-0.38	0.08
	(21.11)	(626.76)	(95.52)	(0.03)
α	0.14	0.14	0.14	0.00
	(99.84)	(2238.70)	(10.37)	(0.00)
β	0.0487	0.0495	0.0872	0.9278
	(0.78)	(1001.00)	(16.30)	(0.03)
θ	0.10	0.10	0.15	0.03
	(6.95)	(340.28)	(17.54)	(0.11)
κ	0.63	0.64	0.72	0.03
	(984.01)	(4693.00)	(124.69)	(0.04)
υ	8.53	9.47	6.67	8.67
	(313.48)	(7898.40)	(1234.20)	(1.63)
Diagnostics				
Log-likelihood	3092.00	2768.60	2968.20	4542.90
Variance persistence	0.19	0.19	0.23	0.93
L-B(20) p-value	0.02	0.00	0.00	0.00
Absolute L-B(20) p-value	0.00	0.00	0.00	0.00
Empirical skewness	-0.30	-0.17	0.12	0.18
Model skewness	1.30	1.25	1.66	0.08
Empirical excess kurtosis	4.68	4.11	4.96	6.90
Model excess kurtosis	6.25	5.79	8.96	4.29

Appendix.B Diebold-Mariano tests of whole data set

We put the Diebold-Mariano tests of the cross-section data set. The four factors results are shown separately. The DOL factor are shown first. Then, we present the Diebold-Mariano tests of HML. The test results of MOM and VAL factors are shown in the last two tables.

Table 11 – DOL Diebold-Mariano tests with whole data set

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of DOL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first five rows correspond to dynamic copula forecasts and the linear correlation benchmark and the orthogonal bench mark with the GARCH models. The next five rows give the forecasting results from dynamic copula models and the linear correlation bench mark and the orthogonal bench mark with the NAGARCH models.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	-0.62	-0.79	-0.29	-0.47	-1.16	-1.18	-1.15	-1.16	-1.16
GCH-t-dcc	0.62	NaN	-2.58	0.55	0.52	-0.69	-0.67	-0.67	-0.69	-0.70
GCH-skt-dcc	0.79	2.58	NaN	0.70	0.68	-0.60	-0.57	-0.57	-0.60	-0.61
GCH-bm-dcc	0.29	-0.55	-0.70	NaN	-1.61	-1.17	-1.19	-1.15	-1.17	-1.17
GCH-orth-dcc	0.47	-0.52	-0.68	1.61	NaN	-1.16	-1.18	-1.15	-1.16	-1.16
NGCH-n-dcc	1.16	0.69	0.60	1.17	1.16	NaN	0.73	1.09	0.76	-0.86
NGCH-t-dcc	1.18	0.67	0.57	1.19	1.18	-0.73	NaN	-0.51	-0.73	-0.76
NGCH-skt-dcc	1.15	0.67	0.57	1.15	1.15	-1.09	0.51	NaN	-1.13	-1.01
NGCH-bm-dcc	1.16	0.69	0.60	1.17	1.16	-0.76	0.73	1.13	NaN	-0.84
NGCH-orth-dcc	1.16	0.70	0.61	1.17	1.16	0.86	0.76	1.01	0.84	NaN

Table 12 – HML Diebold-Mariano tests with whole data set

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of HML factor returns. The table has the same structure as Table 11.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	3.22	3.11	-4.93	-4.92	1.32	1.39	1.39	1.33	1.32
GCH-t-dcc	-3.22	NaN	1.40	-3.38	-3.35	-1.03	-0.80	-0.76	-1.01	-1.03
GCH-skt-dcc	-3.11	-1.40	NaN	-3.28	-3.25	-1.14	-0.92	-0.88	-1.12	-1.14
GCH-bm-dcc	4.93	3.38	3.28	NaN	2.72	1.54	1.59	1.59	1.55	1.54
GCH-orth-dcc	4.92	3.35	3.25	-2.72	NaN	1.51	1.56	1.56	1.52	1.51
NGCH-n-dcc	-1.32	1.03	1.14	-1.54	-1.51	NaN	1.02	0.92	1.11	-1.26
NGCH-t-dcc	-1.39	0.80	0.92	-1.59	-1.56	-1.02	NaN	0.54	-1.01	-1.02
NGCH-skt-dcc	-1.39	0.76	0.88	-1.59	-1.56	-0.92	-0.54	NaN	-0.91	-0.93
NGCH-bm-dcc	-1.33	1.01	1.12	-1.55	-1.52	-1.11	1.01	0.91	NaN	-1.13
NGCH-orth-dcc	-1.32	1.03	1.14	-1.54	-1.51	1.26	1.02	0.93	1.13	NaN

Table 13 – MOM Diebold-Mariano tests with whole data set

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of MOM factor returns. The table has the same structure as Table 11.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	4.04	4.11	5.57	5.21	0.27	0.74	0.87	0.37	0.35
GCH-t-dcc	-4.04	NaN	-2.11	-4.03	-4.02	-1.57	-1.27	-1.17	-1.50	-1.51
GCH-skt-dcc	-4.11	2.11	NaN	-4.10	-4.09	-1.53	-1.23	-1.13	-1.47	-1.48
GCH-bm-dcc	-5.57	4.03	4.10	NaN	4.65	0.25	0.72	0.86	0.35	0.33
GCH-orth-dcc	-5.21	4.02	4.09	-4.65	NaN	0.23	0.70	0.84	0.33	0.31
NGCH-n-dcc	-0.27	1.57	1.53	-0.25	-0.23	NaN	2.37	2.26	2.61	2.59
NGCH-t-dcc	-0.74	1.27	1.23	-0.72	-0.70	-2.37	NaN	1.92	-2.30	-2.31
NGCH-skt-dcc	-0.87	1.17	1.13	-0.86	-0.84	-2.26	-1.92	NaN	-2.19	-2.20
NGCH-bm-dcc	-0.37	1.50	1.47	-0.35	-0.33	-2.61	2.30	2.19	NaN	-2.59
NGCH-orth-dcc	-0.35	1.51	1.48	-0.33	-0.31	-2.59	2.31	2.20	2.59	NaN

Table 14 – VAL Diebold-Mariano tests with whole data set

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of VAL factor returns. The table has the same structure as Table 11.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	-10.57	-10.40	6.66	6.81	-2.70	-2.52	-2.61	-2.70	-2.71
GCH-t-dcc	10.57	NaN	11.46	10.41	10.44	-2.03	-1.80	-1.91	-2.02	-2.04
GCH-skt-dcc	10.40	-11.46	NaN	10.23	10.27	-2.10	-1.88	-1.99	-2.10	-2.11
GCH-bm-dcc	-6.66	-10.41	-10.23	NaN	-5.86	-2.73	-2.55	-2.64	-2.73	-2.74
GCH-orth-dcc	-6.81	-10.44	-10.27	5.86	NaN	-2.72	-2.54	-2.63	-2.72	-2.73
NGCH-n-dcc	2.70	2.03	2.10	2.73	2.72	NaN	4.67	4.88	3.09	-5.05
NGCH-t-dcc	2.52	1.80	1.88	2.55	2.54	-4.67	NaN	-4.44	-4.74	-4.69
NGCH-skt-dcc	2.61	1.91	1.99	2.64	2.63	-4.88	4.44	NaN	-5.00	-4.89
NGCH-bm-dcc	2.70	2.02	2.10	2.73	2.72	-3.09	4.74	5.00	NaN	-3.97
NGCH-orth-dcc	2.71	2.04	2.11	2.74	2.73	5.05	4.69	4.89	3.97	NaN

Appendix C: Diebold-Mariano tests of developed countries

We show the Diebold-Mariano tests for developed countries. The results for the four factors are shown separately. The DOL factor is shown first. We then report the Diebold-Mariano tests for the HML factor. Finally, we report the results for MOM and VAL factors in the last two tables.

Table 15 – DOL Diebold-Mariano tests with developed countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of DOL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first five rows correspond to dynamic copula forecasts and the linear correlation benchmark and the orthogonal bench mark with the GARCH models. The next five rows give the forecasting results from dynamic copula models and the linear correlation bench mark and the orthogonal bench mark with the NAGARCH models.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	-1.56	-1.89	1.18	0.60	-4.35	-3.47	-3.59	-4.36	-4.40
GCH-t-dcc	1.56	NaN	-2.64	1.55	1.54	-4.15	-3.24	-3.35	-4.16	-4.20
GCH-skt-dcc	1.89	2.64	NaN	1.88	1.87	-4.05	-3.12	-3.24	-4.06	-4.10
GCH-bm-dcc	-1.18	-1.55	-1.88	NaN	-1.64	-4.35	-3.48	-3.59	-4.36	-4.40
GCH-orth-dcc	-0.60	-1.54	-1.87	1.64	NaN	-4.35	-3.48	-3.59	-4.36	-4.40
NGCH-n-dcc	4.35	4.15	4.05	4.35	4.35	NaN	6.34	6.34	-7.14	-7.79
NGCH-t-dcc	3.47	3.24	3.12	3.48	3.48	-6.34	NaN	-6.33	-6.35	-6.41
NGCH-skt-dcc	3.59	3.35	3.24	3.59	3.59	-6.34	6.33	NaN	-6.34	-6.41
NGCH-bm-dcc	4.36	4.16	4.06	4.36	4.36	7.14	6.35	6.34	NaN	-7.94
NGCH-orth-dcc	4.40	4.20	4.10	4.40	4.40	7.79	6.41	6.41	7.94	NaN

Table 16 – HML Diebold-Mariano tests with developed countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of HML factor returns. This table has the same structure as Table 15.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	1.24	0.95	1.90	2.25	-3.60	-5.27	-5.81	-3.63	-3.62
GCH-t-dcc	-1.24	NaN	0.01	-1.16	-1.23	-4.86	-6.92	-7.59	-4.91	-4.90
GCH-skt-dcc	-0.95	-0.01	NaN	-0.88	-0.94	-5.24	-7.43	-8.15	-5.28	-5.27
GCH-bm-dcc	-1.90	1.16	0.88	NaN	-1.83	-3.70	-5.40	-5.94	-3.74	-3.73
GCH-orth-dcc	-2.25	1.23	0.94	1.83	NaN	-3.61	-5.29	-5.82	-3.65	-3.64
NGCH-n-dcc	3.60	4.86	5.24	3.70	3.61	NaN	-42.79	-56.08	-17.94	-31.55
NGCH-t-dcc	5.27	6.92	7.43	5.40	5.29	42.79	NaN	-18346.58	43.88	42.98
NGCH-skt-dcc	5.81	7.59	8.15	5.94	5.82	56.08	18346.58	NaN	57.82	56.56
NGCH-bm-dcc	3.63	4.91	5.28	3.74	3.65	17.94	-43.88	-57.82	NaN	8.42
NGCH-orth-dcc	3.62	4.90	5.27	3.73	3.64	31.55	-42.98	-56.56	-8.42	NaN

Table 17 – MOM Diebold-Mariano tests with developed countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of MOM factor returns. This table has the same structure as Table 15.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	1.45	1.17	-2.21	0.95	-6.11	-5.81	-5.72	-6.12	-6.12
GCH-t-dcc	-1.45	NaN	0.39	-1.47	-1.46	-6.10	-5.82	-5.75	-6.11	-6.12
GCH-skt-dcc	-1.17	-0.39	NaN	-1.18	-1.17	-6.07	-5.79	-5.72	-6.08	-6.08
GCH-bm-dcc	2.21	1.47	1.18	NaN	1.68	-6.11	-5.80	-5.72	-6.12	-6.12
GCH-orth-dcc	-0.95	1.46	1.17	-1.68	NaN	-6.11	-5.81	-5.72	-6.12	-6.12
NGCH-n-dcc	6.11	6.10	6.07	6.11	6.11	NaN	6.98	6.96	-7.60	-7.46
NGCH-t-dcc	5.81	5.82	5.79	5.80	5.81	-6.98	NaN	6.81	-7.01	-7.00
NGCH-skt-dcc	5.72	5.75	5.72	5.72	5.72	-6.96	-6.81	NaN	-6.98	-6.98
NGCH-bm-dcc	6.12	6.11	6.08	6.12	6.12	7.60	7.01	6.98	NaN	-6.85
NGCH-orth-dcc	6.12	6.12	6.08	6.12	6.12	7.46	7.00	6.98	6.85	NaN

Table 18 – VAL Diebold-Mariano tests with developed countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of VAL factor returns. This table has the same structure as Table 15.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	-0.07	0.10	-0.67	-0.16	-2.89	-2.76	-2.75	-2.86	-2.86
GCH-t-dcc	0.07	NaN	0.90	0.03	0.07	-2.93	-3.90	-3.87	-2.86	-2.81
GCH-skt-dcc	-0.10	-0.90	NaN	-0.13	-0.10	-3.09	-3.77	-3.76	-3.02	-2.98
GCH-bm-dcc	0.67	-0.03	0.13	NaN	0.57	-2.83	-2.70	-2.69	-2.82	-2.81
GCH-orth-dcc	0.16	-0.07	0.10	-0.57	NaN	-2.89	-2.77	-2.75	-2.87	-2.86
NGCH-n-dcc	2.89	2.93	3.09	2.83	2.89	NaN	-1.27	-1.04	1.24	0.88
NGCH-t-dcc	2.76	3.90	3.77	2.70	2.77	1.27	NaN	2.06	1.28	1.26
NGCH-skt-dcc	2.75	3.87	3.76	2.69	2.75	1.04	-2.06	NaN	1.05	1.03
NGCH-bm-dcc	2.86	2.86	3.02	2.82	2.87	-1.24	-1.28	-1.05	NaN	0.44
NGCH-orth-dcc	2.86	2.81	2.98	2.81	2.86	-0.88	-1.26	-1.03	-0.44	NaN

Appendix D: Diebold-Mariano tests of developing countries

We display the Diebold-Mariano tests for developing countries. The results for the four factors are shown separately. The DOL factor is shown first. We then report the Diebold-Mariano tests for the HML factor. Finally, we report the results for MOM and VAL factorsa in the last two tables.

Table 19 – DOL Diebold-Mariano tests with developing countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of DOL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first five rows correspond to dynamic copula forecasts and the linear correlation bench mark and the orthogonal bench mark with the GARCH models. The next five rows give the forecasting results from dynamic copula models and the linear correlation bench mark and the orthogonal bench mark with the NAGARCH models.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	0.51	-0.01	-0.19	-0.80	-1.09	1.18	0.63	-0.57	-0.60
GCH-t-dcc	-0.51	NaN	-0.61	-0.51	-0.52	-1.09	1.18	0.63	-0.57	-0.60
GCH-skt-dcc	0.01	0.61	NaN	0.01	0.00	-1.09	1.18	0.63	-0.57	-0.60
GCH-bm-dcc	0.19	0.51	-0.01	NaN	-0.98	-1.09	1.18	0.63	-0.57	-0.60
GCH-orth-dcc	0.80	0.52	0.00	0.98	NaN	-1.09	1.18	0.63	-0.57	-0.60
NGCH-n-dcc	1.09	1.09	1.09	1.09	1.09	NaN	1.32	1.39	2.78	2.78
NGCH-t-dcc	-1.18	-1.18	-1.18	-1.18	-1.18	-1.32	NaN	-1.27	-1.27	-1.27
NGCH-skt-dcc	-0.63	-0.63	-0.63	-0.63	-0.63	-1.39	1.27	NaN	-1.03	-1.05
NGCH-bm-dcc	0.57	0.57	0.57	0.57	0.57	-2.78	1.27	1.03	NaN	-2.64
NGCH-orth-dcc	0.60	0.60	0.60	0.60	0.60	-2.78	1.27	1.05	2.64	NaN

Table 20 – HML Diebold-Mariano tests with developing countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of HML factor returns. This table has the same structure as Table 19.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	3.79	3.75	4.60	4.72	1.59	1.43	1.38	1.59	1.57
GCH-t-dcc	-3.79	NaN	3.44	-3.68	-3.78	1.01	0.87	0.82	1.01	1.00
GCH-skt-dcc	-3.75	-3.44	NaN	-3.66	-3.74	0.82	0.69	0.64	0.82	0.81
GCH-bm-dcc	-4.60	3.68	3.66	NaN	-4.53	1.52	1.36	1.31	1.52	1.50
GCH-orth-dcc	-4.72	3.78	3.74	4.53	NaN	1.58	1.43	1.38	1.59	1.57
NGCH-n-dcc	-1.59	-1.01	-0.82	-1.52	-1.58	NaN	-0.40	-0.55	-0.32	-0.03
NGCH-t-dcc	-1.43	-0.87	-0.69	-1.36	-1.43	0.40	NaN	-1.21	0.39	0.45
NGCH-skt-dcc	-1.38	-0.82	-0.64	-1.31	-1.38	0.55	1.21	NaN	0.54	0.61
NGCH-bm-dcc	-1.59	-1.01	-0.82	-1.52	-1.59	0.32	-0.39	-0.54	NaN	0.02
NGCH-orth-dcc	-1.57	-1.00	-0.81	-1.50	-1.57	0.03	-0.45	-0.61	-0.02	NaN

Table 21 – MOM Diebold-Mariano tests with developing countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of MOM factor returns. This table has the same structure as Table 19.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	4.93	4.50	5.47	5.40	-0.87	-0.55	0.18	0.74	-0.94
GCH-t-dcc	-4.93	NaN	3.79	-4.86	-4.83	-0.87	-0.56	0.17	0.74	-0.94
GCH-skt-dcc	-4.50	-3.79	NaN	-4.43	-4.40	-0.87	-0.56	0.16	0.74	-0.94
GCH-bm-dcc	-5.47	4.86	4.43	NaN	5.22	-0.87	-0.55	0.18	0.74	-0.94
GCH-orth-dcc	-5.40	4.83	4.40	-5.22	NaN	-0.87	-0.55	0.18	0.74	-0.94
NGCH-n-dcc	0.87	0.87	0.87	0.87	0.87	NaN	0.82	0.90	0.94	-0.51
NGCH-t-dcc	0.55	0.56	0.56	0.55	0.55	-0.82	NaN	2.93	0.80	-0.93
NGCH-skt-dcc	-0.18	-0.17	-0.16	-0.18	-0.18	-0.90	-2.93	NaN	0.72	-0.97
NGCH-bm-dcc	-0.74	-0.74	-0.74	-0.74	-0.74	-0.94	-0.80	-0.72	NaN	-1.32
NGCH-orth-dcc	0.94	0.94	0.94	0.94	0.94	0.51	0.93	0.97	1.32	NaN

Table 22 – VAL Diebold-Mariano tests with developing countries

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler [2017], over the out-of-sample period from March 11, 2005, to March 20, 2020, for ten different forecasting models of VAL factor returns. This table has the same structure as Table 19.

.	GCH-n-dcc	GCH-t-dcc	GCH-skt-dcc	GCH-bm-dcc	GCH-orth-dcc	NGCH-n-dcc	NGCH-t-dcc	NGCH-skt-dcc	NGCH-bm-dcc	NGCH-orth-dcc
GCH-n-dcc	NaN	2.49	2.69	-2.70	-2.69	-0.19	0.18	1.38	-0.10	0.14
GCH-t-dcc	-2.49	NaN	-2.15	-2.51	-2.53	-0.20	0.18	1.37	-0.11	0.14
GCH-skt-dcc	-2.69	2.15	NaN	-2.70	-2.71	-0.19	0.18	1.37	-0.11	0.14
GCH-bm-dcc	2.70	2.51	2.70	NaN	-2.68	-0.19	0.18	1.38	-0.10	0.14
GCH-orth-dcc	2.69	2.53	2.71	2.68	NaN	-0.19	0.18	1.38	-0.10	0.15
NGCH-n-dcc	0.19	0.20	0.19	0.19	0.19	NaN	0.24	1.63	2.75	2.72
NGCH-t-dcc	-0.18	-0.18	-0.18	-0.18	-0.18	-0.24	NaN	0.60	-0.21	-0.15
NGCH-skt-dcc	-1.38	-1.37	-1.37	-1.38	-1.38	-1.63	-0.60	NaN	-1.61	-1.56
NGCH-bm-dcc	0.10	0.11	0.11	0.10	0.10	-2.75	0.21	1.61	NaN	2.70
NGCH-orth-dcc	-0.14	-0.14	-0.14	-0.14	-0.15	-2.72	0.15	1.56	-2.70	NaN